

$$\frac{d\omega}{d\alpha_R} = \frac{D_1 R_5}{\Delta} < 0, \quad (C.26)$$

$$\frac{d\omega}{d\rho} = \frac{D_1 R_6}{\Delta} < 0, \quad (C.27)$$

$$\frac{d\omega}{dg_N} = \frac{D_1 R_7}{\Delta} > 0, \quad (C.28)$$

$$\frac{d\omega}{d\alpha_Q^h} = \frac{-D_2 R_1}{\Delta} < 0, \quad (C.29)$$

$$\frac{d\omega}{d\alpha_Q^f} = \frac{-D_3 R_1}{\Delta} > 0. \quad (C.30)$$

The signs of equations (C.23) through (C.30) prove Proposition 2.

C.2.3 Proposition 3

Using the system of equations given by (C.21), I establish the comparative steady-state results for the PEG specification regarding $\tilde{\theta}$. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (C.21) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\tilde{\theta}}{dN^f} = \frac{R_2}{\Delta} < 0, \quad (C.31)$$

$$\frac{d\tilde{\theta}}{dN^h} = \frac{R_3}{\Delta} > 0, \quad (C.32)$$

$$\frac{d\tilde{\theta}}{dk} = \frac{R_4}{\Delta} > 0, \quad (C.33)$$

$$\frac{d\tilde{\theta}}{d\alpha_R} = \frac{R_5}{\Delta} > 0, \quad (C.34)$$

ESSAYS ON ENDOGENOUS TECHNOLOGICAL PROGRESS



By

IORDANIS PETSAS

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
2002

Copyright 2002

by

Iordanis Petsas

This dissertation is dedicated to my mother, Stavroula. Without her love and support I would not have started my studies. During my graduate studies, while seriously ill, she always supported me in an extraordinary way and encouraged me to finish my dissertation. I dedicate this work to her with all my love.

ACKNOWLEDGMENTS

My biggest heartfelt thanks go to my mother, Stavroula, who provided me with continuous encouragement. To my deepest sorrow, my mother passed away while I was working on this dissertation. I am extremely sorry that she can not have the chance to see my achievement.

In writing this dissertation, I have benefited from the guidance, encouragement, and knowledge of many people and I offer them my thanks. I would like to thank my parents Stavroula and Nikos Petsas and the rest of my family, Panagiotis and Maria, for supporting my studying abroad and patiently waiting during the long spells of separation.

I am deeply indebted and grateful to Elias Dinopoulos for his guidance and encouragement at every stage of the dissertation. Elias has contributed immensely to my development as an individual, teacher, and researcher. I am very grateful for the role that he has played so well as my great motivator and advisor. I also wish to recognize David Figlio, Douglas Waldo, and Steven Slutsky for their helpful comments and advice. Their keen eyes for detail and their astute comments always came at the right time.

I would also like to thank Panos Hatzipanayotou, my former undergraduate teacher and advisor at the Aristotle University of Thessaloniki, who made me realize the excitement of macroeconomics. He assured me that it is truly worthwhile to pursue a Ph.D. in economics. I am also grateful to Manolis Loukakis, George Papachristou, Nikos Varsakelis, and Chrisoula Zacharopoulou for their dedicated teaching.

My life in Gainesville would have been miserable had I not had wonderful friends. I greatly acknowledge the friendship of Huseyin Yildirim and Hsiu-Chuan Yeh. I also thank Sofia Vidalis for her support and for sharing my path despite her own challenges as a Ph.D. student. This dissertation is dedicated to my mother, who has been the best teacher of my life.

TABLE OF CONTENTS

	<u>page</u>
ACKNOWLEDGMENTS	iv
ABSTRACT	x
CHAPTER	
1 INTRODUCTION	1
1.1 Process versus Product Innovation and Firm Size	1
1.2 General Purpose Technologies and Schumpeterian Growth	3
1.3 Sustained Comparative Advantage and Relative Wages	6
2 PROCESS VERSUS PRODUCT INNOVATION IN MULTIPRODUCT FIRMS	9
2.1 Introduction	9
2.2 A Model of Process versus Product Innovation	13
2.2.1 Demand for Differentiated Products	13
2.2.2 The Duopoly Model	15
2.3 Model of Product versus Process Innovation Considering the R&D Costs	19
2.4 Implications and Extensions	24
3 THE DYNAMIC EFFECTS OF GENERAL PURPOSE TECHNOLOGIES ON SCHUMPETERIAN GROWTH	30
3.1 Introduction	30
3.2 The Model	39
3.2.1 Industry Structure	39
3.2.2 Diffusion of a New GPT	40
3.2.3 Households	42
3.2.4 Product Markets	44
3.2.5 R&D Races	45
3.2.6 Labor Market	49
3.3 Long-Run Equilibrium	49
3.4 Transitional Dynamics	54
3.4.1 Stock Market Behavior	57
3.4.2 Aggregate Investment	61
3.5 Concluding Remarks	62

4 SUSTAINED COMPARATIVE ADVANTAGE IN A MODEL OF SCHUMPETERIAN GROWTH WITHOUT SCALE EFFECTS	70
4.1 Introduction.....	70
4.2 The Model.....	78
4.2.1 Household Behavior.....	79
4.2.2 Product Markets.....	81
4.2.3 R&D Races	83
4.2.4 Labor Markets.....	89
4.3 Steady-State Equilibrium Under the PEG Specification	90
4.4 Comparative Steady-State Analysis Under the PEG Specification	94
4.5 Steady-State Equilibrium Under the TEG Specification	99
4.6 Comparative Steady-State Analysis Under the TEG Specification	102
4.7 Conclusions.....	106
5 CONCLUSION.....	122
APPENDIX	
A PROOFS OF PROPOSITIONS IN CHAPTER 2.....	127
B SUMMARY OF THE HISTORICAL EVOLUTION OF SEMICONDUCTOR TECHNOLOGY AND PROOFS OF PROPOSITIONS IN CHAPTER 3	134
C PROOFS OF PROPOSITIONS IN CHAPTER 4.....	149
REFERENCES	164
BIOGRAPHICAL SKETCH	169

LIST OF FIGURES

<u>Figure</u>	<u>page</u>
2-1 Marginal Profits from Process and Product Innovation Considering R&D Costs as Sunk.....	26
2-2 Net Marginal Profits from Process and from Product Innovations.....	27
2-3 Effect of an Increase in R&D Cost from Product Innovation on n^*_k	28
2-4 Effect of an Increase in R&D Cost from Process Innovation on n^*_k	29
3-1 Steady-State Equilibria.....	64
3-2 Stability of the Balanced-Growth Equilibrium.....	65
3-3 Time Path of the Per Capita Consumption Expenditure After a GPT Arrives in the Economy.....	66
3-4 Time Path of the Market Interest Rate After a GPT Arrives in the Economy.....	66
3-5 Evolution of the Aggregate Investment During the Diffusion Path.....	67
3-6 Effects of a GPT on the Schumpeterian Growth Rate	68
3-7 Effects of a GPT on the Stock Market.....	69
4-1 Steady-State Equilibrium Under the PEG Specification.....	111
4-2 Ranking of the Global R&D Investment, I , Between Home and Foreign, Under the PEG Specification.....	112
4-3 Effects of an Increase in the Foreign Country's Relative Size or in the Population Growth Rate (Under the PEG Specification).....	113
4-4 Effects of an Increase in the R&D Difficulty Parameter, k , in the Unit R&D Labor Requirement or in the Consumer's Subjective Discount Rate (Under the PEG Specification).....	114
4-5 Effects of an Increase in the Foreign's Unit Labor Requirement in Manufacturing (Under the PEG Specification).....	115

4-6	Effects of an Increase in Home's Unit Labor Requirement in Manufacturing (Under the PEG Specification).....	116
4-7	Steady-State Equilibrium Under the TEG Specification.....	117
4-8	Effects of an Increase in the Foreign County's Relative Size in the Consumer's Subjective Discount Rate or in the R&D Difficulty Growth Parameter (Under the TEG Specification).....	118
4-9	Effects of an Increase in the Size of Innovations or in the Population Growth Rate (Under the TEG Specification).....	119
4-10	Effects of an Increase in Foreign's Unit Labor Requirement in Manufacturing (Under the TEG Specification).....	120
4-11	Effects of an Increase in Home's Unit Labor Requirement in Manufacturing (Under the TEG Specification).....	121

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

ESSAYS ON ENDOGENOUS TECHNOLOGICAL PROGRESS

By

Iordanis Petsas

August 2002

Chair: Elias Dinopoulos

Major Department: Economics

This dissertation analyzes several issues involving the economics of endogenous technological progress:

- The effects of firm size on the choice of R&D effort between process and product innovation
- The transitional and long-run dynamic effects of general-purpose technologies on Schumpeterian growth in a closed economy setting
- The interactions among endogenous technological change, the pattern of trade, and income distribution within each country

Chapter 2 develops a differentiated-goods duopoly model in which firms engage in Cournot-Nash quantity competition. I showed that an increase in a firm's efforts devoted to product innovation (given that it is in the product R&D regime) increases its incentives to switch from product to process innovation. Once the firm is in the process R&D regime, it will perform process R&D indefinitely.

Chapter 3 builds a quality-ladders model of scale-invariant Schumpeterian growth. The introduction of a new general-purpose technology increases Schumpeterian growth and per-capita R&D investment in the final steady-state. During the transition path from the initial to the final steady-state equilibrium, the measure of industries that adopt the new general-purpose technology increases, consumption per capita falls, and the interest rate increases. The growth rate of the stock market depends negatively on the rate of the general-purpose technology diffusion process and the magnitude of the general-purpose technology-ridden R&D productivity gains; and positively on the rate of population growth. In addition, the model generates transitional growth cycles of per-capita GNP.

Chapter 4 constructs a two-country (Home and Foreign) general equilibrium model of Schumpeterian growth without scale effects. The scale effects property is removed by introducing two distinct specifications in the knowledge production function: the permanent effect on growth (PEG) specification, which allows policy effects on long-run growth; and the temporary effects on growth (TEG) specification, which generates exogenous long-run economic growth. Under the PEG specification, an increase in the rate of population growth raises Home's relative wage and decreases its range of goods exported to Foreign. Under the TEG specification, an increase in the rate of population growth lowers Home's relative wage and increases its range of goods exported to Foreign.

CHAPTER 1 INTRODUCTION

Economic historians have placed great weight on technology as a force of change. The importance of technological progress has been increased dramatically in the last decade. Many macroeconomic studies place technological progress at the center of the growth process. This trend has been triggered by theoretical developments that allow microeconomic aspects of the innovation process to be linked with macroeconomic outcomes.

The search for causes and effects of technological progress has dominated the recent literature in growth and international trade. In this dissertation, I consider these causes and effects in the context of dynamic growth models. Chapter 2 provides an analysis of two types of technological change, (process versus product innovations) and examines the effects of the firm size on the choice of R&D effort between process and product innovation. Chapter 3 analyzes the impacts of a certain type of drastic innovations, termed general-purpose technologies (GPTs) on Schumpeterian growth. Chapter 4 investigates interactions among endogenous technological change, the pattern of trade, and income distribution within each country. Chapter 5 presents general conclusions. The rest of Chapter 1 briefly describes the motivation, scope, and main findings for each chapter of the dissertation.

1.1 Process versus Product Innovation and Firm Size

Numerous studies have tested the hypothesis about the advantages of size for innovative activity. Kamien and Schwartz (1972), Baldwin and Scott (1987) and Scherer

(1980) reviewed the findings concerning the relationship within industries between firm size and R&D effort. Cohen and Klepper (1996) summarized these findings in the form of stylized facts that characterize this relationship within industries between firm R&D effort and firm size. Some of their key findings are as follows:

- Among R&D performing firms, the number of patents and innovations per dollar of R&D decreases with firm size or level of R&D.
- Among all firms, smaller firms account for a disproportionately large number of patents and innovations relative to their size.

Scherer (1991) found that among manufacturing business units considered as a whole, process R&D increases relative to product R&D as the size of the firm increases, with each tenfold increase in business unit sales associated with a highly significant ten-point increase in the percentage of R&D expenditures devoted to process innovation.

Cohen and Klepper (1996) proposed and tested a theory of how firm size conditions the relative amount of process and product innovation undertaken by firms, using patent data. They found that the share of process R&D undertaken by firms indeed rises with firm size within most industries.

The motivation behind Chapter 2 was to examine the findings mentioned in the studies above using a differentiated-goods duopoly model in which firms engage in Cournot-Nash quantity competition. Firms optimally choose to engage either in product or in process innovation.

The analysis generates several insights. An increase in the number of goods produced by a firm and thus an increase in its size causes the firm to perform process R&D regardless of the R&D regime in which it was located originally. Given that the

firm starts with product R&D, as the number of goods produced increases, the firm's incentives to switch from product to process innovation increase. Once the firm is in the process R&D regime, it continues to perform process R&D indefinitely. This constitutes then a cost-based mechanism that links firm size to the type of R&D activity the firm develops, which is not clear in the models mentioned earlier.

1.2 General Purpose Technologies and Schumpeterian Growth

The unusual combination of more rapid growth and slower inflation in the 1990s initiated a strenuous debate among economists about whether improvements in America's economic performance can be sustained. Many believe that the behavior of information technology provides the key to the recent surge in economic growth. Jorgenson (2001) provides plenty of evidence that the rise in structural productivity growth in the late 1990s in the United States can be traced to the introduction of personal computers and semiconductors, which constitutes the necessary building blocks for the information technology revolution. His estimates indicate that the importance of information technology has steadily increased in the United States over the last 50 years. He estimates that the contribution of information technology to the average growth rate of 3.46% for the last 50 years is 0.4%. The information-technology products contribute 0.5 percentage points to total factor productivity growth for 1995 to 1999, compared to 0.25 percentage points for 1990 to 1995. This reflects the accelerating decline in relative price changes resulting from shortening the product cycle for semiconductors.

Information technology is considered a drastic innovation. In recent years, the literature on these types of drastic innovations has been initiated by Bresnahan and Trajtenberg (1995), who coined the term, "General Purpose Technologies" (GPTs henceforth). A drastic innovation qualifies as a GPT if it has the potential for pervasive

use in a wide range of sectors in ways that drastically change their modes of operation.

To quote from Bresnahan and Trajtenberg (1995):

[Most GPTs play the role of "enabling technologies," opening up new opportunities rather than offering complete, final solutions. For example, the productivity gains associated with the introduction of electric motors in manufacturing were not limited to a reduction in energy costs. The new energy sources fostered the more efficient design of factories, taking advantage of the newfound flexibility of electric power. Similarly, the users of micro-electronics benefit from the surging power of silicon by wrapping around the integrated circuits their own technical advances. This phenomenon involves what we call "innovational complementarities,"....These complementarities magnify the effects of innovation in the GPT, and help propagate them throughout the economy.]

Bresnahan and Trajtenberg (1995, p. 84).

Even though the distinction between a drastic technological innovation and an incremental one is needed to understand the proper roles of technological innovations as engines of growth, economists have paid relatively less attention to the former.

Several empirical studies have documented the cross-industry pattern of diffusion for a number of GPTs (For example, Helpman and Trajtenberg, 1998*b*). In addition, a strand of empirical literature has established that the cross-industry diffusion pattern of GPTs is similar to the diffusion process of product-specific innovations and it is governed by standard *S*-curve dynamics (Griliches, 1957; Jovanovic and Rousseau, 2001). These studies imply that the internal-influence epidemic model can provide an empirically-relevant framework to analyze the dynamic effects of a GPT. During this diffusion process, these drastic innovations could generate growth fluctuations and even business cycles (Schumpeter, 1939, 1950).

Building an analytical model that incorporates all these key characteristics of GPTs and examining their dynamic effects on Schumpeterian growth constitutes one motivation of Chapter 3. The other motivation is to formalize relationships among GPTs, stock market, and R&D investment that were empirically investigated by Jovanovic and

Rousseau (2001) using 114 years of U.S. stock market data. Therefore, I incorporate the presence of a GPT into the standard quality-ladder framework of Schumpeterian growth without scale effects that was developed by Dinopoulos and Segerstrom (1999), and explore the impacts of GPTs on Schumpeterian growth, the stock market, and R&D investment. It is a dynamic model of endogenous growth with endogenous determination of innovation. Firms engage in R&D to discover higher-quality products.

In the presence of a GPT, the model has two steady-state equilibria: the initial steady-state in which no industry has adopted the new GPT, and the final steady-state in which all industries have adopted the new GPT and its diffusion process has been completed. At the final steady-state relative to the initial steady-state: the long-run growth rate is higher even though the per capita consumption expenditure is lower, due to higher aggregate investment.

There are two additional findings of the model. First, there exists a unique globally stable-saddle-path along which the measure of industries that adopt the new GPT increases, the per capita consumption expenditure decreases, the market interest rate increases, and the innovation rate of the industries that have adopted the new GPT decreases at a higher rate than that of those that have not adopted the new GPT. Second, the model exhibits transitional growth cycles of per capita GNP as a consequence of S-curve dynamics.

The introduction of positive population growth in Aghion and Howitt's (1998b) model of GPTs will make these growth cycles shorter and shorter as the size of the economy increases, and in the long-run the GPT-induced cycles disappear. In the model developed in Chapter 3, the fall in output comes from the reduction in per capital

consumption expenditure on final goods and the rise in the per capita R&D investment. As the size of the economy increases (as a result of positive population growth) the duration of the per capita GNP cycle remains the same. When all industries have adopted the new GPT and the diffusion process has been completed, the economy experiences a higher per capita income constant growth rate.

In addition, the growth rate of the stock market depends negatively on the rate of GPT diffusion process and the magnitude of the GPT-ridden R&D productivity gains, and positively on the rate of population growth.

Finally, the effect of the GPT diffusion on the aggregate investment during the adoption process is ambiguous. In the initial stages of the diffusion process, only a limited number of industries adopt the new GPT. These industries are called the early adopters. As more industries adopt the new GPT, the aggregate investment increases.

1.3 Sustained Comparative Advantage and Relative Wages

Many models of endogenous growth and trade emphasize the role of continual product innovation based on R&D investment in determining the pattern of trade between countries (Grossman and Helpman, (1991a, b, c), Taylor (1993)). All these studies exhibit the scale effect property: if one incorporates population growth in these models, then the size of the economy (scale) increases exponentially over time, R&D resources grow exponentially, and so does the long-run growth rate of per-capita real output.

The scale effects property is a consequence of the assumption that the growth rate of knowledge is directly proportional to the level of resources devoted to R&D. Jones (1995a) argued that the scale effects property of earlier endogenous growth models is inconsistent with post-war time series evidence from all major advanced countries that shows an exponential increase in R&D resources and a more-or-less constant rate of

per-capita GDP growth.

Building a quality-ladders model of scale-invariant Schumpeterian growth and examining interactions among endogenous technological change, the pattern of trade, and income distribution within each country constitutes the motivation of Chapter 4. It constructs a two-country (Home and Foreign) general equilibrium model. The scale effects property is removed by introducing two distinct specifications in the knowledge production function: The permanent effects on growth (PEG) specification which allows policy effects on long-run growth; and the temporary effects on growth (TEG) specification, which generates exogenous long-run economic growth.

My approach borrows from Taylor's work (1993) in that industries differ in production technologies. In his model, industries also differ in research technologies and in the set of technological opportunities available for each industry. The absence of heterogeneity in research technologies in my model makes the removal of scale effects more tractable, but eliminates the need for trade in R&D services between countries.

The main point of Chapter 4 is that the removal of scale effects affects the comparative static properties of the model. Furthermore, the various comparative steady-state results depend on the way that the scale effects property is removed. Under the PEG specification, changes in the size of innovations do not affect Home's comparative advantage and its relative wage, while under the TEG specification, an increase in the size of innovations reduces Home's relative wage and raises its comparative advantage.

Under the PEG specification, changes in the size of innovations do not affect relative wages and comparative advantage, while under the TEG specification, an

increase in the size of innovations reduces Home's relative wage and raises its comparative advantage. In addition, under the PEG specification, an increase in the rate of population growth raises Home's relative wage and decreases its range of goods exported to Foreign. On the other hand, under the TEG specification, an increase in the rate of population growth lowers Home's relative wage and increases its range of goods exported to Foreign.

CHAPTER 2 PROCESS VERSUS PRODUCT INNOVATION IN MULTIPRODUCT FIRMS

2.1 Introduction

There is heterogeneity among firms in the degree of product and process innovation in which they engage. The percentage of total R&D dedicated to different types of innovative activity differs greatly across industries. For example, in petroleum refining, almost three-quarters of total R&D is dedicated to process innovation, whereas less than one-quarter of pharmaceutical R&D is dedicated to process innovation. A large part of such differences is due to differences in exogenous industry-level conditions that systematically differentiate the returns to one sort of innovative activity versus another. Link (1982), for example, finds that greater product complexity increases the fraction of effort dedicated to process innovation.

There is an extensive empirical literature in industrial organization, investigating at the industry level the relationship between firm size and the composition of R&D effort, and hence, the nature of innovation. However, inadequate attention seems to have been paid to modeling these two types of R&D activities and trying to support the empirical papers.

The findings of Link (1982) all suggest that within industries, firm size, and, thus, across industries, market structure, may also influence the composition of R&D. Scherer (1991) finds that among manufacturing business units considered as a whole, process R&D increases relative to product R&D as the size of the firm increases, with each

tenfold increase in business unit sales associated with a highly significant ten point increase in the percentage of R&D expenditures devoted to process innovation. Cohen and Klepper (1996) proposed and tested a theory of how firm size conditions the relative amount of process and product innovation undertaken by firms. Their theory explains the close and often proportional relationship within industries between firm size and innovative activity. Their model generates predictions about the relationship between firm size and the share of process R&D. They also tested these predictions using patent data that distinguish between process and product innovation and business unit sales data from the Federal Trade Commission's Line of Business Program. They found that the share of process R&D undertaken by firms indeed rises with firm size within most industries.

One critical question arises about how technologically progressive industries evolve from birth through maturity. When industries are new, there is a lot of entry, firms offer many different versions of the industry's product, the rate of product innovation is high, and market shares change rapidly. Despite continued market growth, subsequently entry slows, exit overtakes entry and there is a shakeout in the number of producers, the rate of product innovation and the diversity of competing versions of the product decline, increasing effort is devoted to improving the production process, and market shares stabilize. This evolutionary pattern has come to be known as the product life cycle (PLC). While numerous papers have contributed to this description, perhaps the most influential one has been that of Abernathy and Utterback's (1978). In their paper, they stress that when a product is introduced, there is considerable uncertainty about user preferences and the technological means of satisfying them. As a result, many firms producing

different variants of the product enter the market and competition focuses on product innovation. As users experiment with the alternative versions of the product and producers learn about how to improve the product, opportunities to improve the product are depleted and a defacto product standard, dubbed a dominant design, emerges.

Producers who are unable to produce efficiently the dominant design exit, contributing to a shakeout in the number of producers. The depletion of opportunities to improve the product coupled with locked-in of the dominant design leads to a decrease in product innovation. This in turn reduces producers' fears that investments in the production process will be rendered obsolete by technological change in the product. Consequently, they increase their attention to the production process and invest more in capital-intensive methods of production, which reinforces the shakeout of producers by increasing the minimum efficient size firm.

Klepper (1996) summarized regularities concerning how entry, exit, market structure, and innovation vary from the birth of technologically progressive industries through maturity. He developed a model emphasizing differences in firm innovative capabilities and the importance of firm size in appropriating the returns from innovation. His model predicts that over time firms devote more effort to process innovation but the number of firms and the rate and diversity of product innovation eventually wither.

One of the goals in this chapter is to examine these findings (within industries the fraction of total R&D a firm devotes to process R&D will be an increasing function of the firm's size) using a differentiated-goods duopoly model in which firms engage in Cournot-Nash quantity competition (in contrast to Klepper's model (1996), in which all

firms produce a standard product). Firms optimally choose to engage either in product or in process innovation.

The present model differs from the ones in the studies mentioned above in a number of respects. First, I consider multi-product firms that produce a number of differentiated goods in a duopoly setting, and investigate the relationship between firm size and R&D activity based on demand and cost functions. Second, labor is the only primary factor of production: it can be used to produce the differentiated goods and R&D services. R&D services result in discoveries of better production techniques, which enhance the productivity of labor employed in the manufacturing of the differentiated goods. R&D product services result in discoveries of new goods (adding or improving product features).

The analysis generates several insights. For example, an increase in the number of goods produced by a firm and thus an increase in its size (since in my model, the firm size is measured by the firm's sales and the firm's sales are proportional to the number of goods produced) causes the firm to perform process R&D regardless the R&D regime in which it is located originally. Given that the firm starts with product R&D, as the number of goods produced increases, the firm's incentives to switch from product to process innovation increase. Once the firm is in the process R&D regime, it continues to perform process R&D indefinitely. This constitutes a cost-based mechanism that links firm size to the type of R&D activity the firm develops, which is not clear in the models mentioned earlier.

This chapter is organized as follows. Section 2.2 develops the model, and shows how it can explain Klepper's finding. Section 2.3 investigates the relationship between

firm size and innovative activity in different regimes. Section 2.4 considers the implications of the analysis for the relationship between firm size and the types of innovative activities undertaken by firms within industries as well as extensions of the model are considered. The algebraic details and proofs of propositions in this chapter are relegated to Appendix A.

2.2 A Model of Process versus Product Innovation

2.2.1 Demand for Differentiated Products

In this section, I develop a model to explain the influence of firm size on the effort devoted to process relative to product innovation. I imagine an identifiable sectoral structure of commodities. Thus, a pencil is a well-defined object and so are a refrigerator, a personal computer, a restaurant meal, and a haircut. Each one of these goods is a differentiated product, however, in the sense that there are many varieties of it available in the market and many more varieties that could potentially be produced. There are red and yellow pencils, soft and hard pencils, white and green refrigerators, 16MB memory personal computers and 128 MB memory personal computers and so on.

Since products can be differentiated in many dimensions, one way to introduce preferences for differentiated products is to assume that there are commodities that individuals like to consume in many varieties, so that variety is valued in its own right. The tastes of a representative individual are represented by the following utility function (the economy is able to produce a large number of products, all of which enter symmetrically into demand):

$$U = \sum_{i=1}^N D_i^{\alpha} + x_0, \quad 0 < \alpha < 1, \quad (1)$$

where D_i is consumption of the i^{th} product, N is the number of available products and x_0 is an outside good.¹

The demand for any individual product i can be derived by solving the following maximization problem:

$$\max_{D_1, D_2, \dots, D_N} U$$

$$\text{subject to } \sum_{i=1}^N D_i P_i = I,$$

where I is the individual's income.

By forming the lagrangian function (L), the above problem is equivalent to:

$$\max_{D_1, D_2, \dots, D_N} L = \max_{D_1, D_2, \dots, D_N} \left[\sum_{i=1}^N D_i + x_0 + \lambda \left(I - \sum_{i=1}^N D_i P_i - x_0 \right) \right],$$

where λ is the lagrangian multiplier.

The first order conditions to the above problem are the following:

$$\frac{\partial L}{\partial D_i} = 0 \Leftrightarrow \alpha D_i^{\alpha-1} - \lambda P_i = 0, \quad \forall i = 1, \dots, N.$$

$$\frac{\partial L}{\partial \lambda} = 0 \Leftrightarrow x_0^* = I - \sum_{i=1}^N P_i D_i^*,$$

$$\frac{\partial L}{\partial x_0} = 0 \Leftrightarrow \lambda = 1.$$

Since λ is fixed, the inverse demand functions for the differentiated goods are²:

¹ See more on this in Dixit and Stiglitz (1977).

² In the monopolistic competition models, λ is taken as fixed because it is assumed that the number of goods produced is large and thus each firm's pricing policy has a negligible effect on the marginal utility of income.

$$P_i = \alpha D_i^{\alpha-1}, \quad \forall i = 1, \dots, N. \quad (2)$$

where D_i is the quantity demanded per capita. As the total number of consumers is fixed, I can set population at 1 without loss of generality. In this case, I do not have to distinguish between total and per-capita quantities, so I let D_i ($i = 1, \dots, N$) denote the respective (total or per capita) quantities of the differentiated goods.

2.2.2 The Duopoly Model

I consider a duopoly in which each firm produces a range of differentiated goods. Good i is produced with the following production function:

$$X_{ik} = \mu^{q_i} L_{ik}, \quad \forall i = 1, \dots, n_1 \text{ if } k = 1, \text{ and } \forall i = n_1 + 1, \dots, N \text{ if } k = 2, \quad (3)$$

where k denotes firm, X_{ik} denotes the output of good i produced by firm k , and L_{ik} denotes the amount of labor used in the production of good i by firm k . Thus, production functions are the same for all product varieties within each firm. The number of potential varieties is assumed to be countably infinite, so that only a finite subset of the range is actually produced. The parameter μ (>1) is the quality increment per innovation, whereas the parameters q_1 and q_2 represent the number of innovations undertaken by firm 1 and 2 respectively ($q \in \{0, 1, 2, \dots\}$). The total number of goods available in the economy is $n_1 + n_2 = N$. Production of each product variety in such an industry will be undertaken by only one producer, since the other firm can always do better by introducing a new product variety than by sharing in the production of an existing product type.

Using labor as the numeraire, I can normalize the wage rate to unity. Then, the marginal cost (the cost per unit) of good X_{i1} is $1/\mu^{q_1}$ ($\forall i = 1, \dots, n_1$), when firm 1 knows how to produce X_{i1} with the q_1 th process. The marginal cost of good X_{i2} is

$1/\mu^{q_2}$ ($\forall i = 1, \dots, n_2$), when firm 2 knows how to produce X_{i2} with the q_2 th process. I

assume that both firms produce the quantities demanded of each good. Since each firm is a multi-product firm, I measure the firm size by their sales. For example, sales for firm 1 are given by:

$$\sum_{i=1}^{n_1} P_i D_i = nPD.$$

The above equation implies that the prices are the same for all goods produced within each firm and the quantities produced are the same within each firm. Since the sales of each firm are proportional to the number of goods produced by that firm, I can use that number to measure the firm size.

In this section, I consider R&D costs as sunk (that is they already have been incurred and they are fixed). The profit function for firm 1 for producing n_1 goods is as follows:

$$\pi_1 = \sum_{i=1}^{n_1} P_i D_i - \frac{1}{\mu^{q_1}} \sum_{i=1}^{n_1} D_i. \quad (4)$$

In the same way, the profit function for firm 2 for producing n_2 goods, is given by:

$$\pi_2 = \sum_{i=n_1+1}^N P_i D_i - \frac{1}{\mu^{q_2}} \sum_{i=n_1+1}^N D_i. \quad (5)$$

By differentiating equations (4) and (5), with respect to the quantities (D_i), I obtain the quantities produced of each good and then I can obtain the value profit functions. Since the value profit functions are symmetric for both firms, I can generalize them and use k to denote either firm ($k=1,2$). By doing some comparative statics with respect to n_k and q_k , I obtain the following two equations and Proposition 1:

$$\Delta \pi_{k,q_k}^* = \pi_k^*(n_k + 1, q_k) - \pi_k^*(n_k, q_k) = \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) \left(\frac{1}{\mu^{q_k}} \right)^{a/(a-1)} > 0, \quad (6)$$

where $\Delta \pi_{k,q_k}^*$ is the marginal profit of firm k from performing product innovation.

$$\Delta \pi_{k,q_k}^* = \pi_k^*(q_k + 1, n_k) - \pi_k^*(q_k, n_k) = n_k \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) \mu^{q_k a/(1-\alpha)} (\mu^{a/(1-\alpha)} - 1) > 0, \quad (7)$$

where $\Delta \pi_{k,q_k}^*$ is the marginal profit of firm k from performing process innovation and

where k denotes firm ($k=1,2$).

Proposition 1. *Total profits for firm k ($k=1,2$) depend positively on the number of goods produced and the number of process innovations it performs.*

Proof. See Appendix A.

Equations (6) and (7) provide a basis for explaining why the firm size tends to increase the marginal returns from process R&D, but not the marginal returns from product R&D. In equation (6), the marginal returns to product R&D are independent of the firm size, whereas in equation (7) the marginal returns to process R&D depend positively on the firm size.

I can now solve explicitly for the number of goods (n_k^*), at which the firm is indifferent between performing one more product R&D and one more process R&D. By equalizing equations (6) and (7), I obtain the following proposition:

Proposition 2. *If $n_k^* = \left(1/\mu^{q_k} \right)^{a/(a-1)} \left(1/\mu^{a q_k/(1-\alpha)} (\mu^{a/(1-\alpha)} - 1) \right)$, then firm k is indifferent between performing one more product innovation and performing one more process innovation.*

Proof. See Appendix A.

Proposition 2 provides a rationale for Scherer's (1991) finding that larger firms devote a greater fraction of their R&D to process innovation. The intuition behind this result is straightforward. The marginal returns to process R&D rise with the firm's size (as it is measured with n_k) while the marginal returns to product R&D are constant (independent of n_k). If $n_k^* > \left((1/\mu^s)^{n/(s-1)} \left((1/\mu^{as/(1-s)}) (\mu^{n/(1-s)} - 1) \right) \right)$, then it is more profitable for firm k ($=1,2$) to switch from product to process innovation.

Figure 2-1 shows the relationship between the marginal profits from process and from product innovation considering the R&D costs as sunk costs. The marginal profit from process innovation is an increasing function of the number of goods produced, while the marginal profit from product innovation is independent of the number of goods produced.

Point A in Figure 2-1 reflects a situation, in which the firm is indifferent between performing one more process innovation and one more product innovation. For a small number of goods produced (any point to the left of point A), the firm performs only product R&D. The reason is that, when a firm develops a product innovation, it reaches new buyers as well as raise price given some degree of transient monopoly power.

For a large number of goods produced (any point to the right of point A), the firm performs only process innovation. The reason is that, when a firm develops a process innovation, it lowers its average cost. Thus, the firm will increase its profits by the decrease in average cost times the level of output of each good produced. Since, in the model, whenever a firm develops a process innovation, it applies to all goods, the larger the number of goods produced the greater the increase in the firm's profits.

Next, I consider the case, where the firm k is in the product R&D regime (any point to the left of point A in Figure 2-1). I examine the effect of developing one more product innovation on the incentives to switch from product to process innovation. Firm k is in the product R&D regime if and only if

$$\Delta\pi_{k,q_k}^* - \Delta\pi_{k,n_k}^* = \frac{1-\alpha}{\alpha^{(n+1)/(1-\alpha)}} \left(\mu^{a/(1-\alpha)} - 1 \right) \mu^{a q_k / (1-\alpha)} \left(n_k (\mu^{a/(1-\alpha)} - 1) - 1 \right) < 0, \quad (8)$$

$$\text{or } n_k^* < 1 / (\mu^{a/(1-\alpha)} - 1).$$

By differentiating equation (8) with respect to n_k , and after rearranging, I obtain:

$$\Delta(\Delta\pi_{k,q_k}^* - \Delta\pi_{k,n_k}^*)_{n_k} = \mu^{a q_k / (1-\alpha)} \left(\frac{1-\alpha}{\alpha^{(n+1)/(1-\alpha)}} \right) \left(\mu^{a/(1-\alpha)} - 1 \right) > 0, \quad (9)$$

since $\mu > 1$.

Proposition 3. *Given that the firm k ($=1,2$) is in the product R&D regime, as each firm produces more goods (develops more product innovations), its incentives to switch from product to process innovations increase.*

Proof. See Appendix A.

Proposition 3 explains the relationship between the firm size and the type of innovative activity. The intuition behind this result is straightforward. The returns to process R&D rise proportionally with the firm size while the returns to product R&D are constant with the firm size. Consequently, an increase in the number of goods produced must have a positive effect on process than product R&D, causing the firm to switch from product to process R&D.

2.3 Model of Product versus Process Innovation Considering the R&D Costs

In this section, I examine how firms choose optimally their R&D effort between process and product innovation. I assume for simplicity that firms develop either product

or process R&D, but not both of them at the same time. There are three regimes: product R&D regime (in the area to the left of point A in Figure 2-1), process R&D regime (in the area to the right of point A in Figure 2-1) and neutral R&D regime (point A in Figure 2-1). Firms start with product R&D because of monopoly power and then they switch to process R&D to exploit economies of scale.

In this section, I examine the decision of the firm to perform one more product innovation or to perform one more process innovation for a given number of goods produced and processes developed and given that the firm is in the process R&D regime (any point to the right of point A in Figure 2-1).

This decision is based on the difference between the net marginal profit from product innovations and the net marginal profit from process innovations. Net marginal profit from process innovation includes the R&D cost for process innovation and net marginal profit from product innovation includes the R&D cost for product innovation. If the net marginal profit from process is larger than the net marginal profit from product innovations, then it is more profitable for the firm to switch from product to process innovations.

I assume that firms spend money on product R&D (R_p) and process R&D (R_c). Let's denote with R_{pk} the amount spent on product R&D for good i by firm k and R_{ck} the amount spent on process R&D for all goods by firm k . I assume that process R&D applies to all goods, that is it reduces equally the average cost of all goods produced. I further assume that the amount spent on product R&D is the same for all goods produced within the firm.

Utilizing the assumptions mentioned above, I arrive at the following proposition:

Proposition 4.

If $n_k^* = 1/(\mu^{a/(1-a)} - 1) - \left(\alpha^{\frac{a+1}{(a-1)}} (R_{pk} - R_{ck}) / (1-\alpha) \mu^{\frac{a\alpha k}{(1-a)}} (\mu^{\frac{a}{(1-a)}} - 1) \right)$, then firm k is

indifferent between performing one more product innovation and performing one more process innovation.

Proof. See Appendix A.

Figure 2-2 depicts the net marginal profits from process and from product innovations. At point A, firm k produces n_k^* , and it is indifferent between performing one more product innovation and performing one more process innovation.

Now, by differentiating n_k^* in Proposition 3 with respect to R_{pk} and R_{ck} , I obtain the following propositions:

Proposition 5. *The number of goods produced by firm k is a decreasing function of the firm's R&D cost on product innovation.*

Proof. See Appendix A.

Intuitively, as the firm's R&D cost on product innovation is higher, the net marginal profit from product R&D decreases. Thus, at the original number of goods produced at which the firm is indifferent between performing one more product and one more process innovation, the net marginal profit from process innovation is greater than the net marginal profit from product innovation causing the firm to produce fewer goods.

Figure 2-3 shows the effect of an increase in the R&D cost from product innovation (R_{pk}) on n_k^* . When R_{pk} increases, the net marginal profit from product R&D line shifts down and the equilibrium moves from point A to point B. At point B, the number of goods produced at which the firm is indifferent between developing one more

product innovation and developing one more process innovation is less than that of point A ($n_k^* < n_k^*$).

Proposition 6. *The number of goods produced by firm k is an increasing function of the firm's R&D cost on process innovation.*

Proof. See Appendix A.

As the firm's R&D cost on process innovation increases, the net marginal profit from process innovation decreases. Thus, at the original number of goods produced at which the firm is indifferent between performing one more product and one more process innovation, the net marginal profit from process innovation is lower than the net marginal profit from product innovation causing the firm to produce more goods.

Figure 2-4 shows the effect of the increase in the R&D cost from process innovation (R_{ck}) on n_k^* . When R_{ck} increases, the net marginal profit from process innovation shifts to the right and the equilibrium moves from point A to point B. At point B, the number of goods produced at which the firm is indifferent between developing one more product innovation and developing one more process innovation is higher than that of point A ($n_k^* > n_k^*$).

Next, I consider the case, where the firm is at a point to the right of point A in the Figures, so that it performs only process innovations, since the net marginal profit from process is greater than the net marginal profit from product innovations in this area.

The firm k is in the process R&D regime if and only if

$$\Delta\pi_{k,qk}^* - \Delta\pi_{k,nk}^* = \frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \left(\mu^{a/(1-\alpha)} - 1 \right) \mu^{aq_k/(1-\alpha)} \left(n_k \left(\mu^{a/(1-\alpha)} - 1 \right) - 1 \right) > 0.$$

This last expression implies the following:

$$[n_k^* (\mu^{\alpha/(1-\alpha)} - 1) - 1] > 0. \quad (10)$$

Suppose the firm has already developed n_k product innovations and q_k process innovations. Given (n_k, q_k) , and given the fact that the firm is performing only process innovations, I examine if it is more profitable for the firm to develop one more process innovation ($q_k + 1$) or to develop one more product innovation ($n_k + 1$).

By developing one more process innovation, the difference between the marginal profit from process and the marginal profit from product innovation increases by:

$$\Delta(\Delta\pi_{k,q_k}^* - \Delta\pi_{k,n_k}^*)_{q_k} = \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) (\mu^{\alpha/(1-\alpha)} - 1) \mu^{aq_k/(1-\alpha)} [n_k (\mu^{\alpha/(1-\alpha)} - 1) - 1], \quad (11)$$

where $\Delta(\Delta\pi_{k,q_k}^* - \Delta\pi_{k,n_k}^*)_{q_k}$ is the change in the difference between the marginal profit from process and the marginal profit from product innovation due to change in the number of process innovations by one.

If firm k is in the process R&D regime, equation (10) holds and implies that the sign of equation (11) is positive. That is, the firm has an incentive to continue performing process R&D. By developing one more product innovation, the difference between the marginal profit from process and the marginal profit from product innovation increases by:

$$\Delta(\Delta\pi_{k,q_k}^* - \Delta\pi_{k,n_k}^*)_{n_k} = \mu^{aq_k/(1-\alpha)} \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) (\mu^{\alpha/(1-\alpha)} - 1) > 0. \quad (12)$$

The signs of equations (11) and (12) imply:

Proposition 7. *Given that firm k ($=1,2$) is in the process R&D regime, it will continue to perform process R&D indefinitely.*

Proof. See Appendix A.

This finding suggests that large firms have no incentives to do product R&D, reinforcing the conclusion of earlier studies on R&D effort and firm size (i.e., it supports the idea of R&D cost spreading). The fraction of process R&D versus product R&D rises monotonically with firm size. There is a critical point where the firm enters the process R&D regime and its incentives to remain in this regime are high. Once the firm performs only process R&D, its incentives to switch to product R&D disappear. That is, the firm will find more profitable to find ways to produce its goods cheaper than to create higher quality goods or more variety.

2.4 Implications and Extensions

The notion that the firm size and the choice between process and product innovation follow a common pattern has become part of the folklore. My findings support the basic idea that larger firms have an advantage in R&D because of the larger output over which they can apply the results-and thus spread the costs-of their R&D (Cohen and Klepper (1996)). Note, however, that once a firm switches from product to process, it continues to develop only process (since the size of process innovation in my model is greater than one). If the size of process innovation were small, by performing one more product R&D in the process R&D regime would increase the incentives to switch from process to product innovation. Indeed, if the size of process innovation is less than one, process innovation is not profitable.

Two limitations of the model need to be brought to the forefront. First, my conclusions depend on and are limited by the functional forms and the assumptions of my model. Another production function, for example, can make the model conform better to the real world. That is, once a firm is in the process R&D regime, after a critical number of process innovations, it can be possible to switch from process to product innovations.

Second, the assumption that firms will not attend to the production process until product innovation has slowed sufficiently is also restrictive. Yet the history of the automobile industry and others, such as tires and antibiotics, indicates that great improvements were made in the production process well before the emergence of any kind of dominant design. Indeed, many of these improvements were based on human and physical improvements that were not rendered obsolete by subsequent major product innovations. One possible extension of the model is to relax this assumption and explore the implications of this change on firm's optimal decision on choosing the fraction of process and product innovation. This relaxation would change some of the results, but it would also complicate the analysis of the model.

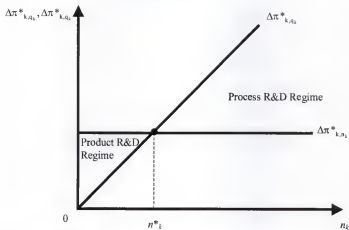


Figure 2-1. Marginal Profits from Process and Product Innovation Considering R&D Costs as Sunk

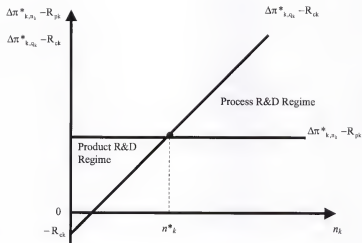


Figure 2-2. Net Marginal Profits from Process and from Product Innovations

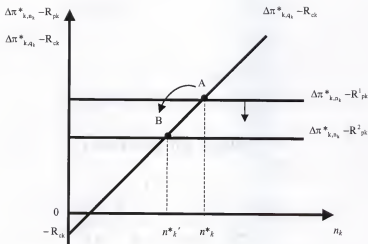


Figure 2-3. Effect of an Increase in R&D Cost from Product Innovation on n^*_k

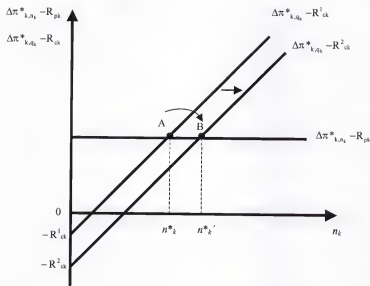


Figure 2-4. Effect of an Increase in R&D Cost from Process Innovation on n_k^*

CHAPTER 3

THE DYNAMIC EFFECTS OF GENERAL PURPOSE TECHNOLOGIES ON SCHUMPETERIAN GROWTH

3.1 Introduction

In any given economic “era” there are major technological innovations, such as electricity, the transistor, and the Internet, that have far-reaching and prolonged impact. These drastic innovations induce a series of secondary incremental innovations. The introduction of the transistor, for example, triggered a sequence of secondary innovations, such as the development of the integrated circuit and the microprocessor, which are also considered drastic innovations.³ These main technological innovations are used in a wide range of different sectors inducing further innovations. For example, microprocessors are now used in many everyday products like telephones, cars, personal computers, and so forth.

In general, drastic innovations have three key characteristics. The first feature refers to the generality of purpose, i.e., drastic innovations affect a wide range of industries and activities within each industry. Consequently, Bresnahan and Trajtenberg (1995) christened these types of drastic innovations “General Purpose Technologies” (GPTs henceforth). Several empirical studies have documented the cross-industry pattern

³ In Appendix B, I provide a summary of the historical evolution of semiconductor technology, from the birth of the transistor in 1947 invented by the Bell Labs to the development of the microprocessor in 1971 by Intel. As a special case, I elaborate on the computer and the Internet to show how these important drastic innovations are based on a previous drastic innovation, the transistor.

of diffusion for a number of GPTs.⁴ In addition, a strand of empirical literature has established that the cross-industry diffusion pattern of GPTs is similar to the diffusion process of product-specific innovations and it is governed by standard *S*-curve dynamics.⁵ In other words, the internal-influence epidemic model can provide an empirically-relevant framework to analyze the dynamic effects of a GPT.⁶ During this diffusion process, these drastic innovations could generate growth fluctuations and even business cycles.

Schumpeter provides details on the connection between drastic innovations and business cycles:

⁴ For example, Helpman and Trajtenberg (1998a) provide evidence for the diffusion of the transistor. They state that transistors were first adopted by the hearing aids industry. Later, transistors were used in radios followed by the computer industry. These three industries are known as early adapters of the transistor GPT. The fourth sector to adopt the transistor was the automobile industry, followed by the telecommunication sector.

⁵ For example, Griliches (1957) studied the diffusion of hybrid seed corn in 31 states and 132 crop-reporting areas among farmers. His empirical model generates an *S*-curve diffusion path. Andersen (1999) confirmed the *S*-shaped growth path for the diffusion of entrepreneurial activity, using corporate and individual patents granted in the U.S. between 1890 and 1990. Jovanovic and Rousseau (2001) provide more evidence for an *S*-shaped curve diffusion process by matching the spread of electricity with that of personal computer use by consumers.

⁶ As Paul Saffo, founder and director of the Institute for the Future, in Palo Alto, California, in his interview, *Surfing the "S" Curve*, in the *News Link 1st Quarter* (2001) edition (Volume 9, No.1), describes the *S*-curve dynamics for Internet: "In terms of the Internet, we're at the bottom of the *S*-curve. Everything we've seen so far is just the beginning." The vertical rise in the *S*-curve will depend on new technologies that are going to advance in the future such as fiber optics, 3D chip arrays, DNA computing, nanocomputers, and much more. For more information on emerging technologies see "Taking Advantage of Technological Acceleration: Tracking Emerging Technologies and Trends" by Manyworlds (2001, www.manyworlds.com) and Appendix B.

"It is by no means farfetched or paradoxical to say that "progress" destabilizes the economic world, or that it is by virtue of its mechanism a *cyclical process*."

Schumpeter (1939, p. 138).

"These revolutions periodically reshape the existing structure of industry by introducing new methods of production-the mechanized factory, the electrified factory, chemical synthesis and the like; new commodities, such as railroad service, motorcars, electrical appliances; new forms of organization."

Schumpeter (1950, p. 68).

"Times of innovation...are times of effort and sacrifice, of work for the future, while the harvest comes after...The harvest is gathered under recessive symptoms and with more anxiety than rejoicing...[During] recession...much dead wood disappears."

Schumpeter (1939, p.143).

Second, the dynamic effects of these GPTs take a long period of time to materialize. For instance, David (1990) argues that it may take several decades before major technological innovations can have significant impact on macroeconomic activity.⁷ Third, these GPTs act as "engines of growth". As a better GPT becomes available, it gets adopted by an increasing number of user industries and fosters complementary advances that raise the industry's productivity growth. As the use of a GPT spreads throughout the economy, its effects become significant at the aggregate level, thus affecting overall productivity growth. In his presidential address to the American Economic Association, Jorgenson (2001) documents the role of information technology in the resurgence of U.S. growth in the late 1990s.⁸ There is plenty of evidence that the rise in structural

⁷ David (1990) describes a phase of twenty-five years in the case of the electric dynamo. He argues that the observed productivity slowdown in the earlier stage of electrification and computerization was due to the adjustment process associated with the adoption of a new GPT.

⁸ At the aggregate level, information technology is identified with the output of computers, communications equipment, and software. These products appear in the GDP as investments by businesses, households, and governments along with net exports to the rest of the world.

productivity growth in the late 1990s can be traced to the introduction of personal computers and the acceleration in the price reduction of semiconductors, which constituted the necessary building blocks for the information technology revolution.⁹

The growth effects of GPTs have been analyzed formally by Helpman and Trajtenberg (1998b). In their model GPTs require complementary inputs before they can be applied profitably in the production process. Complementary inputs developed for previous GPTs are not suited for use with a newly arrived GPT. The sequential arrival of GPTs generates business cycles. A typical cycle consists of two phases, a phase where firms produce final goods with the old GPT and components are being developed for the new GPT, and a second phase where final goods producers switch to the new GPT and the development of components for that GPT continues.¹⁰ Output declines in the first phase of a cycle as workers switch from production to research to invent new inputs and increases again in the second phase once the new technology is implemented.¹¹

⁹ Another study from OECD documents that U.S. investment in information processing equipment and software increased from 29% in 1987 to 52% in 1999. The diffusion of information and communication equipment accelerated after 1995 as a new wave of information and communication equipment, based on applications such as the World Wide Web and the browser, spread rapidly throughout the economy.

¹⁰ In the same paper, the authors also provide an extension of their model in which they introduce a continuum of final-good sectors with different degrees of productivity gains from GPTs and with the intermediate inputs being common to all sectors. As a result, a GPT spreads gradually across the economy, first to sectors that stand to gain most from its adoption and last to sectors that stand to gain least. Concerning output growth, it drops right at the beginning of the cycle (as in the base case), but then it exhibits positive growth during phase 1.

¹¹ There is a growing literature with this approach. See, for example, Helpman and Rangel (1998), Aghion and Howitt (1998b) and the volume edited by Helpman (1998).

In this chapter, I analyze formally the effects of a GPT within a state-of-the-art model of Schumpeterian growth without scale effects. Schumpeterian (R&D-based) growth is a type of growth that is generated through the endogenous introduction of new goods or processes based on Schumpeter's (1934) process of creative destruction, as opposed to physical or human-capital accumulation.¹²

Earlier models of Schumpeterian growth assumed that the growth rate of technological change depends positively on the level of R&D resources devoted to innovation at each instant in time. As population growth causes the size of the economy (scale) to increase exponentially over time, R&D resources also grow exponentially, and so does the long-run growth rate of per-capita real output. In other words, long-run Schumpeterian growth in these models exhibits scale effects. Two influential papers by Jones (1995*a, b*) provided time series evidence for the absence of these scale effects. This evidence led theorists to construct Schumpeterian growth models that exclude scale effects.¹³

My approach to modeling GPTs has the following features. First, the model abstracts from scale effects and generates long-run growth, which is consistent with the time-series evidence presented by Jones (1995*a*). Second, I take into consideration the above mentioned evidence on long diffusion lags associated with the adoption of a new GPT. I therefore analyze both the transitional dynamics and long-run effects of a new

¹² There are two classes of scale invariant Schumpeterian growth models; endogenous and exogenous. Endogenous [exogenous] Schumpeterian growth models are those in which *long-run* growth can [cannot] be affected by permanent policy changes.

¹³ Dinopoulos and Thompson (1999) provide a survey of the empirical evidence on scale and growth, and describe recent attempts to develop models that generate growth without scale effects.

GPT. Third, I assume that a GPT is beneficial to all firms in each industry. Thus, when a GPT is implemented in an industry, it affects the productivity of R&D workers, the size of all future innovations in that industry and its growth rate. Finally, I assume that although a GPT's rate of diffusion is exogenous, its diffusion path across a continuum of industries is governed by *S*-curve dynamics.¹⁴

I incorporate the presence of a GPT into the standard quality-ladder framework of Schumpeterian growth without scale effects that was developed by Dinopoulos and Segerstrom (1999). In the model, there is positive population growth and one factor of production, labor. Final consumption goods are produced by a continuum of structurally identical industries. Labor in each industry can be allocated between two economic activities, manufacturing of high-quality goods, and R&D services that are used to discover new products of higher quality.

The arrival of innovations in each industry is governed by a memoryless Poisson process whose intensity depends positively on R&D investments and negatively on the rate of difficulty of conducting R&D. Following Dinopoulos and Segerstrom (1999), I assume that R&D becomes more difficult over time in each industry. Specifically, I assume that the productivity of R&D workers declines as the size of the market (measured by the level of population) increases. This assumption captures the notion that it is more difficult to introduce new products and replace old ones in a larger market.¹⁵

¹⁴ This assumption can be justified by the empirical observation provided by David (1991) that the measure of industries that adopted the dynamo and the number of consumers that have adopted computers follow an *S*-curve.

¹⁵ Several authors have developed microfoundations for this assumption. Young (1998), Dinopoulos and Thompson (1998), and Aghion and Howitt (1998a, Chapter 12) have combined tastes for horizontal and vertical product differentiation to generate models in which absolute levels of R&D drive productivity growth at the firm-level, but aggregate

For any given measure of industries with the new GPT (i.e., in the absence of *S*-curve dynamic diffusion of technology), the model generates a unique steady-state equilibrium. In this equilibrium, per capita consumption expenditure is constant over time, the aggregate stock value increases at the same rate as the constant rate of population growth, and the long-run Schumpeterian growth is endogenous (Proposition 1).

In the absence of a new GPT, the economy does not exhibit zero long-run growth as in previous models of GPTs (see Helpman and Trajtenberg (1998b), Aghion and Howitt (1998b), and Eriksson and Lindh (2000) among others). That is, the long-run growth rate depends positively on the rate of innovation (which equals per capita R&D) and thus, any policy that affects per capita R&D investment has long-run growth effects. In addition, the removal of scale effects allows me to analyze the effects of changes in the rate of growth of population that is absent from earlier models of GPTs.

In the presence of a GPT, the model has two steady-state equilibria: the initial steady-state in which no industry has adopted the new GPT, and the final steady-state in

R&D in larger economies is diffused over a larger number of product lines or industries. At the steady state, the number of varieties is proportional to the level of population. As population grows, the number of varieties increases and aggregate R&D is diffused over a larger number of product lines or industries, making R&D more difficult.

Dinopoulos and Syropoulos (2000) have provided microfoundations for this specification in a model of Schumpeterian growth, where the discovery of higher quality products is modeled as an R&D contest (as opposed to an R&D race) in which challengers engage in R&D and incumbent firms allocate resources to rent-protecting activities. Rent-protecting activities are defined as costly attempts of incumbent firms to safeguard the monopoly rents from their past innovations. These activities can delay the innovation of better products by reducing the flow of knowledge spillovers from incumbents to potential challengers, and/or increase the costs of copying existing products. Their model postulates that R&D may become more difficult as the size of the economy grows because incumbent firms may allocate more resources to rent-protecting activities.

which all industries have adopted the new GPT and its diffusion process has been completed. At the final steady-state relative to the initial steady-state: the long-run growth rate is higher, the aggregate investment is higher, the per capita consumption expenditure is lower, and the market interest rate is equal to the subjective discount rate (Proposition 2).

I use Mulligan and Sala-i-Martin's (1992) time-elimination method to study the transitional dynamics of the model. This analysis generates several additional findings. First, there exists a unique globally stable-saddle-path along which the measure of industries that adopt the new GPT increases, the per capita consumption expenditure decreases, the market interest rate increases, and the innovation rate of the industries that have adopted the new GPT decreases at a higher rate than that of those that have not adopted the new GPT. Second, the model exhibits transitional growth cycles of per capita GNP as a consequence of *S*-curve dynamics (Proposition 3).

The introduction of positive population growth in Aghion and Howitt's (1998b) model of GPTs will make these growth cycles shorter and shorter as the size of the economy increases, and in the long-run the GPT-induced cycles disappear. In the present model, the fall in output comes from the reduction in per capital consumption expenditure on final goods and the rise in the per capita R&D investment. As the size of the economy increases (as a result of positive population growth) the duration of the per capita GNP cycle remains the same. When all industries have adopted the new GPT and the diffusion process has been completed, the economy experiences a higher per capita income constant growth rate.

I also analyze the effects of the adoption of the new GPT on the stock market. The growth rate of the stock market depends negatively on the rate of GPT diffusion process and the magnitude of the GPT-ridden R&D productivity gains, and positively on the rate of population growth. It also follows a U-shaped path during the diffusion process of the new GPT (Proposition 4).

During the transition period, there are two types of industries in the economy: one that has adopted the new GPT and one that has not adopted it yet. The former type of industries is more innovative in terms of discovering higher quality products than the latter type of industries. In the initial stages of a GPT's diffusion, the aggregate stock value decreases, since most of the industries belong to the latter type of industries. As more industries switch to the new GPT, the aggregate stock value rises. This result is consistent with the previous GPT-based models.¹⁶

However, the mechanism that links the growth rate of the stock market with the GPT differs from these models. This mechanism allows the model to identify important factors, such as the rate of the population growth, that affect the stock market, which otherwise would have been ignored. In Helpman and Trajtenberg's (1998b) model, for

¹⁶ Jovanovic and Rousseau (2001) document empirically how technology has affected the U.S. economy over the past century, using 114 years of U.S. stock market data. Their estimates reveal evidence that entries to the stock market as a percentage of firms listed in each year, were proportionately largest between 1915 and 1929, and that these levels were not again approached until the mid-1980s. About half of American households and most businesses were connected to electricity in 1920, and about one half of the households and most businesses today own or use computers. Both expansions, therefore, coincide with periods during which electricity and information technology saw widespread adoption. During times of rapid technological change, the new entrants of the stock market will grab the most value from previous entrants because the incumbents will find hard to keep up. The downward trend in the starting values of the vintages reflects a slowing down in the growth of the stock market.

example, during the first phase, the components of both the best practice GPT and of the previous one have positive value. When the economy is in the second phase of a typical cycle, only components of the best practice GPT are valuable because at that time it is known that no component of older technologies will ever be used. Thus, the introduction of a new GPT brings a sharp decline in the real value of the stock market during a substantial part of phase one, but it picks up toward the end of the phase. In the second phase, the stock market rises.

The effect of the GPT diffusion on the aggregate investment during the adoption process is ambiguous (Proposition 5). In the initial stages of the diffusion process, only a limited number of industries adopt the new GPT. These industries are called the early adopters. As more industries adopt the new GPT, the aggregate investment increases.

The rest of this chapter consists of four parts. Section 3.2 develops the structure of the model. Section 3.3 analyzes the long-run properties of the model and Section 3.4 deals with the transitional dynamics. Section 3.5 summarizes the model's key findings and suggests possible extensions. Appendix B describes the evolution of semiconductor technology. The algebraic details and proofs of propositions are also relegated to Appendix B.

3.2 The Model

3.2.1 Industry Structure

I consider an economy with a continuum of industries indexed by $\theta \in [0, 1]$. In each industry θ , firms are distinguished by the quality j of the products they produce. Higher values of j denote higher quality and j is restricted to taking on integer values. At time $t = 0$, the state-of-the-art quality product in each industry is $j = 0$, that is, some firm

in each industry knows how to produce a $j = 0$ quality product and no firm knows how to produce any higher quality product. To learn how to produce higher quality products, firms in each industry engage in R&D races. In general, when the state-of-the-art quality in an industry is j , the next winner of an R&D race becomes the sole producer of a $j+1$ quality product. Thus, over time, products improve as innovations push each industry up its "quality ladder", as in Grossman and Helpman (1991c).

3.2.2 Diffusion of a New GPT

The diffusion path of a new GPT is modeled as follows: The economy has achieved a steady-state equilibrium, manufacturing final consumption goods with an old GPT. I begin the analysis at time $t = t_0$, when a new GPT arrives unexpectedly. Firms in each industry start adopting the new GPT at an exogenous rate.¹⁷

At each point in time, a fraction of industries, ω , uses the new GPT and a fraction of industries, $(1-\omega)$, does not use the new GPT. For example, if the old GPT is the steam power and the new GPT is electricity, ω industries use electricity in their production and

¹⁷ Aghion and Howitt (1988b) model the spread of GPTs using a continuum of sectors. In their model, the innovation process involves three stages. First, the GPT is discovered. Then each sector discovers a "template" on which research can be based. Finally, that sector implements the GPT when its research results in a successful innovation. They have computed paths of the fraction of sectors experimenting with the new GPT and the fraction using the new GPT and found that the time path of the later follows a logistic curve (S-curve). Thus, they analyze the transitional dynamics of GPTs by endogenizing the arrival rate of GPTs and running simulations. Endogenizing the rate of diffusion in my model would be an interesting avenue for further research, although it would make the analysis of the transitional dynamics complicated without adding much value on the results.

(1- ω) industries use steam power in their production.¹⁸ I use the epidemic model to describe the diffusion of a new GPT across the continuum of industries.¹⁹ Its form can be described by the following differential equation,

$$\frac{\dot{\omega}}{\omega} = \delta(1 - \omega), \quad (1)$$

where $\dot{\omega} = \partial\omega/\partial t$ denotes the rate of change in the fraction of industries that use the new GPT and $\delta > 0$ is the rate of diffusion. Equation (1) states that the number of new adoptions during the time interval dt , $\dot{\omega}$, is equal to the number of remaining potential adopters, $(1-\omega)$, multiplied by the probability of adoption, which is the product of the fraction of industries that have already adopted the new GPT, ω , and the parameter δ , which depends upon factors such as the attractiveness of the innovation and the frequency of adoption, both of which are assumed to be exogenous.

¹⁸ Devine (1983) provides an excellent historical perspective on electrification where he documents the transformation from shafts to wires. He states: "Until late in the nineteenth century, production machines were connected by a direct mechanical link to the power sources that drove them. In most factories, a single centrally located prime mover, such as a water wheel or steam engine, turned iron or steel "line shafts" via pulleys and leather belts...By the early 1890s then, direct current motors had become common in manufacturing, but were far from universal. Mechanical drive was first electrified in industries such as clothing and textile manufacturing and printing, where cleanliness, steady power and speed, and ease of control were critical". Helpman and Trajtenberg (1998a) explore the adoption of the transistor, an important semiconductor GPT by a number of industries. As they state: The early user sectors were hearing aids and computers. The prominent laggards were telecommunications and automobiles. These examples indicate that the timing of adopting a new GPT differs across industries.

¹⁹ See Thirly and Ruttan (1987, pp.77-89) for various applications of the epidemic model to the diffusion of technology.

The solution to equation (1) expresses the measure of industries that have adopted the new GPT as a function of time and yields the equation of the sigmoid (*S*-shaped) logistic curve as follows:

$$\omega = \frac{1}{[1 + e^{-(\gamma + \delta t)}]}, \quad (2)$$

where γ is the constant of integration. Notice that for $t \rightarrow \infty$, equation (2) implies that all industries have adopted the new GPT.²⁰

3.2.3 Households

The economy is populated by a continuum of identical dynastic families that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. The number of members in each family grows over time at the exogenous rate $g_N > 0$. I normalize the measure of families in the economy at time 0 to equal unity. Then the population of workers in the economy at time t is $N(t) = e^{g_N t}$.

Each household is modeled as a dynastic family²¹, which maximizes the discounted utility

$$U = \int_0^{\infty} e^{-(\rho - g_N)t} \log u(t) dt, \quad (3)$$

²⁰ When $t \rightarrow -\infty$, then $\omega = 0$. If one assumes that the new GPT arrives at time $t=0$, then $\omega > 0$. That is, the new GPT is introduced in the economy by a given fraction of industries ω (i.e., the industry or industries that developed this particular GPT).

²¹ Barro and Sala-i-Martin (1995, Ch. 2) provide more details on this formulation of the household's behavior within the context of the Ramsey model of growth.

where $\rho > 0$ is the constant subjective discount rate. In order for U to be bounded, I assume that the effective discount rate is positive (i.e., $\rho - g_N > 0$). Expression $\log u(t)$ captures the per capita utility at time t , which is defined as follows:

$$\log u(t) = \int_0^1 \log \left[\sum_j \lambda(\theta)^j q(j, \theta, t) \right] d\theta. \quad (4)$$

In equation (4), $q(j, \theta, t)$ denotes the quantity consumed of a final product of quality j in industry $\theta \in [0, 1]$ at time t . Parameter $\lambda(\theta)$ measures the size of quality improvements and is equal to

$$\lambda(\theta) = \begin{cases} \lambda_1 & \text{if } \theta \in [0, \omega] \\ \lambda_0 & \text{if } \theta \in [\omega, 1], \end{cases} \quad (5)$$

where $\lambda_1 > \lambda_0 > 1$. At each point in time t , each household allocates its income to maximize equation (4) given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function for the product in each industry with the lowest quality-adjusted price

$$q(j, \theta, t) = \frac{c(t)N(t)}{p(j, \theta, t)}, \quad (6)$$

where $c(t)$ is per capita consumption expenditure, and $p(j, \theta, t)$ is the market price of the good considered. The quantity demanded of all other goods is zero.

Given this static demand behavior, the inter temporal maximization problem of the representative household is equivalent to

$$\max_{c(t)} \int_0^\infty e^{-(\rho - g_N)t} \log c(t) dt, \quad (7)$$

subject to the inter temporal budget constraint $\dot{a}(t) = r(t)a(t) + w(t) - c(t) - g_N a$, where $a(t)$ denotes the per capita financial assets, $w(t)$ is the wage income of the representative household member, and $r(t)$ is the instantaneous rate of return. The solution to this maximization problem obeys the well-known differential equation

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \quad (8)$$

According to equation (8), per capita consumption expenditure would increase over time if the instantaneous interest rate exceeded the consumer's subjective discount rate ρ .

3.2.4 Product Markets

Every firm in each industry θ uses labor $L(\theta, t)$ as the sole input in its production according to the following production function

$$Q(\theta, t) = \frac{L(\theta, t)}{\alpha_Q}, \quad (9)$$

where α_Q is the unit labor requirement. The monopolist engages in limit pricing, i.e., it charges a price equal to unit cost of manufacturing a product times the quality increment

$$P = \lambda(\theta)\alpha_Q w. \quad (10)$$

At each instant in time, the incumbent monopolist produces the state-of-the-art quality product and earns a flow of profits

$$\pi(\theta, t) = \left(\frac{\lambda(\theta) - 1}{\lambda(\theta)} \right) c(t)N(t). \quad (11)$$

3.2.5 R&D Races

Labor is the only input used to do R&D in any industry. Each firm in each industry θ produces R&D services by employing labor $L_R(\theta, t)$ under the constant returns to scale production function²²

$$R(\theta, t) = \frac{\mu(\theta)}{\alpha_R} L_R(\theta, t), \quad (12)$$

In equation (12), $\alpha_R/\mu(\theta)$ is the unit-labor requirement in the production of R&D services and $\mu(\theta)$ is equal to

$$\mu(\theta) = \begin{cases} \mu_1 = \mu & \text{if } \theta \in [0, \omega] \\ \mu_0 = 1 & \text{if } \theta \in [\omega, 1], \end{cases} \quad (13)$$

where $\mu > 1$. A firm k that engages in R&D discovers the next higher-quality product with instantaneous probability $I_k dt$, where dt is an infinitesimal interval of time and

$$I_k(\theta, t) = \frac{R_k(\theta, t)}{X(t)}. \quad (14)$$

$R_k(\theta, t)$ is firm k 's R&D outlays and $X(t)$ captures the difficulty of R&D in a typical industry. I assume that the returns to R&D investments are independently distributed across challengers, across industries, and over time. Therefore, the industry-wide

²² The empirical evidence on returns to scale of R&D expenditure is inconclusive. Diminishing returns would make the analysis of the transitional dynamics more complicated. Segerstrom and Zolnerek (1999) among others developed a model where they allow for diminishing returns to R&D effort at the firm level and industry leaders have R&D cost advantages over follower firms. In their model, when there are diminishing returns to R&D and the government does not intervene both industry leaders and follower firms invest in R&D.

probability of innovation can be obtained from equation (14) by summing up the levels of R&D across all challengers. That is,

$$I(\theta, t) = \sum_k I_k(\theta, t) = \frac{R(\theta, t)}{X(t)}, \quad (15)$$

where $R(\theta, t)$ denotes total R&D services in industry θ . Variable $I(\theta, t)$ is the effective R&D.²³ The arrival of innovations follows a memoryless Poisson process with intensity I_1 for the industries that have adopted the new GPT and I_0 for industries that have not adopted the new GPT.

Early models of Schumpeterian growth considered $X(t)$ to be constant over time. This implied that the rates of innovation and the long-run growth increase exponentially as the scale of the economy grows exponentially. This scale-effects property is inconsistent with post-war time-series evidence presented in Jones (1995a).

A recent body of theoretical literature has developed models of Schumpeterian growth without scale effects.²⁴ Two approaches have offered possible solutions to the scale-effects problem. The first generates exogenous long-run Schumpeterian growth models.²⁵ The second approach generates models that exhibit endogenous long-run

²³ The variable $I(\theta, t)$ is the intensity of the Poisson process that governs the arrivals of innovations in industry θ .

²⁴ See Dinopoulos and Thompson (1999) for an overview of these models.

²⁵ Jones (1995b), and Segerstrom (1998) have removed scale effects by assuming that R&D becomes more difficult over time because "the most obvious ideas are discovered first." The model that results from their specification is called the *temporary effects of growth* (TEG) model. In these models, the growth rate does not depend on any measure of scale. Increases in the steady-state level of R&D raise technology and income per capita at any point in time, but they do not raise the growth rate.

Schumpeterian growth.²⁶ Here I adopt the second approach and remove the scale-effects property by assuming that the level of R&D difficulty is proportional to the market size measured by the level of population,

$$X(t) = kN(t), \quad (16)$$

where $k > 0$ is a parameter.²⁷ Consumer savings are channeled to firms engaging in R&D through the stock market. The assumption of a continuum of industries allows consumers to diversify the industry-specific risk completely and earn the market interest rate. At each instant in time, each challenger issues a flow of securities that promise to pay the flow of monopoly profits defined in (11) if the firm wins the R&D race and zero otherwise. Consider now the stock-market valuation of the incumbent firm in each industry. Let $V(t)$ denote the expected discounted profits of a successful innovator at time t when the monopolist charges a price p for the state-of-the-art quality product. Because each quality leader is targeted by challengers who engage in R&D to discover the next higher-quality product, a shareholder faces a capital loss $V(t)$ if further innovation occurs. The event that the next innovation will arrive occurs with instantaneous probability λdt , whereas the event that no innovation will arrive occurs with instantaneous probability

²⁶ Young (1998), Aghion and Howitt (1998a, Ch.12), Dinopoulos and Thompson (1998), Peretto (1998), and Peretto and Smulders (1998) remove the scale effects property by essentially the same mechanism as the one developed by exogenous Schumpeterian growth models. They introduced the concept of localized intertemporal R&D spillovers. Dinopoulos and Syropoulos (2000) proposed a novel mechanism based on the notion of innovation-blocking activities that removes the scale-effects property and generates endogenous long-run Schumpeterian growth. Their model offers a novel explanation to the observation that the difficulty of conducting R&D has been increasing over time.

²⁷ Informational, organizational, marketing, and transportation costs can readily account for this difficulty. Arroyo, et al. (1995) have proposed this specification under the name of the *permanent effects of growth* (PEG) model, and have provided time-series evidence for its empirical relevance.

1-Idt. Over a time interval dt , the shareholder of an incumbent's stock receives a dividend $\pi(t)dt$ and the value of the incumbent appreciates by $dV(t) = [\partial V(t)/\partial t]dt = \dot{V}(t)dt$. The absence of profitable arbitrage opportunities requires the expected rate of return on stock issued by a successful innovator to be equal to the riskless rate of return r ; that is,

$$\frac{\dot{V}(\theta, t)}{V(\theta, t)}[1 - I(\theta, t)dt] + \frac{\pi(\theta, t)}{V(\theta, t)}dt - \frac{[V(\theta, t) - 0]}{V(\theta, t)}I(\theta, t)dt = rdt. \quad (17)$$

Taking limits in equation (17) as $dt \rightarrow 0$ and rearranging terms appropriately gives the following expression for the value of monopoly profits

$$V(\theta, t) = \frac{\pi(\theta, t)}{r(t) + I(\theta, t) - \frac{\dot{V}(\theta, t)}{V(\theta, t)}}. \quad (18)$$

Consider now the maximization problem of a typical challenger k . This firm chooses the level of R&D investment $R_k(\theta, t)$ to maximize the expected discounted profits

$$V(\theta, t) \frac{R_k(\theta, t)}{X(t)}dt - w \frac{\alpha_k}{\mu(\theta)} R_k(\theta, t)dt, \quad (19)$$

where $I_k dt = [R_k(\theta, t)/X(t)]dt$ is the instantaneous probability it will discover the next higher-quality product and $w \alpha_k R_k(\theta, t)/\mu(\theta)$ is the R&D cost of challenger k .

Free entry into each R&D race drives the expected discounted profits of each challenger down to zero and yields the following equilibrium condition as follows:

$$V(\theta, t) = \frac{w \alpha_k k N(t)}{\mu(\theta)}. \quad (20)$$

3.2.6 Labor Market

All workers are employed by firms in either production or R&D activities. Taking into account that each industry leader charges the same price p and that consumers only buy goods from industry leaders in equilibrium, it follows from (9) that total employment of labor in production is $\int_0^1 Q(\theta, t) d\theta$. Solving (12) for each industry leader's R&D employment $L_R(\theta, t)$ and then integrating across industries, total R&D employment by industry leaders is $\int_0^1 [R(\theta, t) \alpha_R / \mu(\theta)] d\theta$. Thus, the full employment of labor condition for the economy at time t is

$$N(t) = \int_0^1 Q(\theta, t) \alpha_Q d\theta + \int_0^1 \frac{\alpha_R R(\theta, t)}{\mu(\theta)} d\theta. \quad (21)$$

Equation (21) completes the description of the model.

3.3 Long-Run Equilibrium

The dynamic behavior of the economy is governed by two equations that determine the evolution of the per capita consumption expenditure, c , and the number of industries that adopt the new GPT, ω . To facilitate the interpretation and understanding of my results, I begin by deriving expressions for long-run per capita real output and long-run growth. Following the standard practice of Schumpeterian growth models, one can obtain the following deterministic expression for sub-utility $u(t)$, which is appropriately weighted consumption index and corresponds to real per capita income²⁸

$$\log u(t) = \log c - \log \alpha_Q + I(\theta, t) \log \lambda(\theta) - \log \lambda(\theta). \quad (22)$$

²⁸ See Dinopoulos (1994) for an overview on Schumpeterian growth theory.

The economy's long-run Schumpeterian growth is defined as the rate of growth of sub-utility $u(t)$, $g_u = \dot{u}(t)/u(t)$. By differentiating equation (22) with respect to time, I obtain the following:

$$g_u = \frac{\dot{u}(t)}{u(t)} = I(\theta, t) \log \lambda(\theta), \quad (23)$$

which is a standard expression for long-run growth in quality-ladders growth models.

Because the size of each innovation becomes larger (i.e., $\lambda_1 > \lambda_0$) after all industries have adopted the new GPT (i.e., the diffusion process has been completed), long-run growth, g_u , can be affected not only through changes in the rate of innovation, but also through the diffusion of the new GPT.

After substitution of equations (6) and (10) into the first integral of equation (21), the demand for manufacturing labor is given by the following expression:

$$\frac{cN(t)}{w} \left(\frac{\omega}{\lambda_1} + \frac{(1-\omega)}{\lambda_0} \right). \quad (24)$$

Substituting equation (12) into the second integral of equation (21), the demand for R&D labor is given by:

$$\omega \frac{\alpha_R}{\mu} R_1 + (1-\omega) \alpha_R R_0. \quad (25)$$

At this point it is useful to choose labor as the numeraire of the model and setting $w = 1$.

(26)

Combining equations (24) and (25) with equations (15) and (16) and taking into account (26) yields the *resource condition*

$$1 = c \left(\frac{\omega}{\lambda_1} + \frac{(1-\omega)}{\lambda_0} \right) + k \alpha_R \left(\frac{\omega}{\mu} I_1 + (1-\omega) I_0 \right), \quad (27)$$

which defines a negative linear relationship between per capita consumption expenditure, c , and the effective R&D, I . The above resource condition holds at each instant in time because by assumption factor markets clear instantaneously.

I now derive the differential equation that determines the growth rate of per capita consumption expenditure, $\dot{c}/c$, as a function of its level and the rate of innovation.

Equation (20) holds at each instant in time, so it yields $\dot{V}(\theta, t)/V(\theta, t) = \dot{X}(t)/X(t) = g_N$.

In other words, the values of expected discounted profits, $V(t)$, and the level of R&D difficulty, $X(t)$, grow at the constant rate of population growth, g_N . Combining equations (18) and (20) (after substituting equation (11) into equation (18) and equation (26) into equation (20)), I obtain

$$c = \frac{\lambda(\theta)a_R k}{(\lambda(\theta) - 1)\mu(\theta)} [\rho + I(\theta, t) - g_N], \quad (28)$$

which defines a positive linear relationship between per capita consumption expenditure, c , and the effective R&D, I . It also implies the familiar condition that $r = \rho$, which means that the market interest rate must be equal to the subjective discount rate in the steady-state equilibrium. This property is shared by all Schumpeterian models where growth is generated by the introduction of final consumption (as opposed to intermediate production) goods.

Let a hat “^” over variables denote their market value in a steady-state equilibrium. The resource condition (27) and the equilibrium R&D condition (28) determine simultaneously the long-run equilibrium values of per capita consumption expenditure, $\hat{c}$, and the rates of innovation, $\hat{I}_0$ and $\hat{I}_1$. Figure 3-1 illustrates the two steady-state equilibria: the initial steady-state (point A) where no industry has adopted the

new GPT (i.e., $\omega = 0$) and the final steady-state (point B) where all industries have adopted the new GPT (i.e., $\omega = 1$). When $\omega = 0$ the balanced-growth resource condition is

$$l = \frac{c}{\lambda_g} + k\alpha_R l_0, \quad (29)$$

and the balanced-growth R&D condition is given by equation (28) (when $\omega = 0$). The vertical axis measures consumption per capita, c , and the horizontal axis measures the rate of innovation, l . The resource condition is reflected by the negatively-sloped line $N_0 N_0$ and the R&D equilibrium condition is represented by the positively-sloped line $R_0 R_0$. Their unique intersection at point A determines the long-run values $\hat{c}(0)$ and $\hat{l}_0(0)$, where $\hat{c}(0)$ denotes the per-capita consumption expenditure evaluated at $\omega = 0$ and $\hat{l}_0(0)$ denotes the innovation rate for industries that have adopted the new GPT evaluated at $\omega = 0$. Therefore, I arrive at:

Proposition 1. *For a given $\omega \in [0, 1]$, where ω is the measure of industries with a new GPT, there exists a unique steady-state equilibrium such that the long-run Schumpeterian growth, $\hat{g}_a$, is endogenous and does not exhibit scale effects: it depends positively on policies that affect the size of innovations, λ , the labor productivity in R&D services, $\mu(\theta)/\alpha_R$, and the rate of population growth, g_N ; it depends negatively on the consumer's subjective discount rate, ρ . At each steady-state equilibrium, consumption expenditure per capita, $\hat{c}$, is constant, the interest rate, $\hat{r}(t)$, is equal to the constant subjective discount rate, ρ , and the aggregate stock value, $\bar{V}$, increases at the same rate as the constant rate of population growth, g_N .*

Proof. See Appendix B.

The removal of scale effects from the long-run growth rate, g_u , depends on the assumption that the level of R&D difficulty is proportional to the market size. At the steady-state equilibrium, the level of R&D difficulty, $X(t)$, increases exponentially at the rate of population growth g_N (i.e., $\dot{X}(t)/X(t) = g_N$) as can be seen from equation (16). The absence of a new GPT does not result in zero long-run growth rate as in the Helpman and Trajtenberg (1998b), Aghion and Howitt (1998b), and Eriksson and Lindh (2000) models. That is, the long-run growth rate depends positively on per capita R&D and thus, any policy that affects this variable has long-run growth effects. The following proposition describes the long-run properties of the economy:

Proposition 2. *If ω is governed by S-curve dynamics, there are only two steady-state equilibria: the initial steady-state equilibrium arises before the adoption of the new GPT, where $\omega = 0$, and the final steady-state equilibrium is reached after the diffusion process of the new GPT has been completed, where $\omega = 1$. At the final steady-state equilibrium: aggregate investment is higher, $\hat{I}(1) > \hat{I}(0)$, long-run growth rate is higher, $\hat{g}_u(1) > \hat{g}_u(0)$, per capita consumption expenditure is lower, $\hat{c}(1) < \hat{c}(0)$, per-capita stock market valuation of the incumbent in each industry is lower, $\hat{V}(1)/N < \hat{V}(0)/N$, relative to the initial steady-state equilibrium. In both steady states the market interest rate is equal to the subjective discount rate, $\hat{r} = \rho$.*

Proof. See Appendix B.

These comparative steady-state properties can be illustrated with the help of Figure 3-1. Before the introduction of the new GPT, the economy is in a steady state

(point A) where $\omega = 0$, with per capita consumption expenditure $\hat{c}(0)$, and with innovation rate $\hat{I}_0$. An increase in the measure of industries that adopt the new GPT makes the R&D condition in Figure 3-1 shift downward from R_0R_0 (where $\omega = 0$) to R_1R_1 (where $\omega = 1$) and the resource condition shift upward from N_0N_0 to N_1N_1 , resulting in higher long-run rate of innovation and in lower long-run consumption per capita. In other words, when all industries have adopted the new GPT, the long-run Schumpeterian growth rate increases. The new steady state is at point B, where $\omega = 1$, with per capita consumption expenditure $\hat{c}(1)$, and innovation rate $\hat{I}_1$.

3.4 Transitional Dynamics

I analyze the transitional dynamics of the model by adapting the time-elimination method described by Mulligan and Sala-i-Martin (1992).²⁹ The time-elimination method enables me to construct a system of two differential equations that govern the evolution of c and ω . Since equation (28) holds at each instant in time (when the subjective discount rate, ρ , is replaced by the interest rate, r), I can solve for the rates of innovation for the two types of industries, I_0 and I_1 . After substituting these rates into the resource condition (27), which holds at each instant in time, I can solve for the market interest rate along any path and obtain

$$r = \frac{\mu(c-1)}{k\alpha_K[\mu - \omega(\mu-1)]} + g_N. \quad (30)$$

Substituting (30) into (8) yields the following differential equation:

$$\frac{\dot{c}}{c} = r - \rho = \frac{\mu(c-1)}{k\alpha_K[\mu - \omega(\mu-1)]} + g_N - \rho. \quad (31)$$

²⁹ See also Mulligan and Sala-i-Martin (1991) for more details on this method.

Equations (31) and (1) determine the evolution of the two endogenous variables of the model, per capita consumption expenditure, c , and the number of industries that have adopted the new GPT, ω . Since the right-hand side of equation (31) is decreasing in ω , $\dot{c} = 0$ defines the downward-sloping curve in Figure 3-2. Starting from any point on this curve, an increase in ω leads to $\dot{c} > 0$ and a decrease in ω leads to $\dot{c} < 0$. The right-hand side of equation (1) is independent of c , and therefore the $\dot{\omega} = 0$ locus is a vertical line. Starting from any point on this line, decrease in ω leads to $\dot{\omega} > 0$. The area to the left of the vertical line (i.e., locus $\dot{\omega} = 0$) identifies a region in which the potential number of adopters is greater than one. Therefore, this region is not feasible. There exists a downward-sloping saddle path going through the unique balanced-growth equilibrium point B. Thus, I arrive at:

Proposition 3. *Assume that $\delta > (g_N - \rho)$. Then, there exists a unique negative-sloping globally stable-saddle-path going through the final unique balanced-growth equilibrium point B. Along the saddle path, the measure of industries that adopt the new GPT, ω , increases, the per capita consumption expenditure, c , decreases, the market interest rate, r , increases, the innovation rate of the industries that have adopted the new GPT, I_1 , decreases at a higher rate than that of those that have not adopted the new GPT, I_0 . In addition, there exist transitional growth cycles of per-capita GNP.*

Proof. See Appendix B.

The analysis is predicated on the assumption of perfect foresight.³⁰ When the new GPT arrives, per capita consumption expenditure, c , jumps down instantaneously to $\tilde{c}$ (point A' in Figure 3-2). This per capita consumption expenditure jump lowers the interest rate to $\tilde{r}$ (Figure 3-4) since there are more savings available. The downward jumps on the per capital consumption expenditure and on the interest rate imply an upward jump on the innovation rates of both types of industries; those that have adopted the new GPT and those that have not adopted the new GPT ($\tilde{i}_1$ and $\tilde{i}_0$ in Figure 3-5).

Figure 3-1 illustrates that the R&D line R_0R_0 will shift downwards and the resource line N_0N_0 will shift upwards with the arrival of the new GPT resulting in lower per-capita consumption expenditure. Going back to Figure 3-2, the instantaneous decrease in c is reflected by a movement from point A to point A' . The decrease in per-capita consumption expenditure leads to a decrease in the market interest rate r (from equation (30), which always hold). When the market interest rate r is lower than the subjective rate ρ , per capita consumption expenditure decreases even further, until the market interest rate approaches the subjective discount rate at the new steady state (point B in Figures 3.1 and 3.2). During the transition dynamics (i.e., as the equilibrium moves from point A' to point B in Figure 3-2), the interest rate increases leading to more savings and a decrease in per capita consumption expenditure. At point B in Figure 3-2, all industries have adopted the new GPT.

³⁰ There also exists a degenerate equilibrium where the adoption of the new GPT is not completed. Suppose that when a new GPT arrives, every potential consumer expects that no one will decrease their consumption expenditure, in order to finance innovation. As a result, it does not pay to decrease consumption expenditure of a single consumer, because the new GPT will never be fully adopted. In this event, the pessimistic expectations are self-fulfilling, and no new GPTs are fully adopted. I do not discuss these types of equilibria in what follows.

Along the transition path, the aggregate investment may increase or decrease. One possible path of the aggregate investment is shown in Figure 3-5 by the dotted curve. There is an upward jump in the innovation rate of industries that have not adopted the new GPT (from point B_0 to point B in Figure 3-5).

Figures 3-3 and 3-4 show the time paths of per capita consumption expenditure and the market interest rate (where t_0 indicates the time when the new GPT arrives in the economy and t_a indicates the time when all industries in the economy have adopted the new GPT). Figure 3-6 shows the effect of a GPT on the Schumpeterian growth rate. The adoption of the new GPT entails cyclical growth patterns.³¹ The growth rate decreases in the initial stages of the adoption of the new GPT. There exist transitional growth cycles.

3.4.1 Stock Market Behavior

The fact that the adoption of a new GPT affects positively the productivity of R&D together with free entry into each R&D race are the key factors in explaining the behavior of the stock market. The probability of discovering the next higher quality product in each industry increases with the adoption of the new GPT and so does the probability that the incumbent in each industry will be replaced by a follower firm (i.e., the hazard rate). This link between the GPT adoption and higher risk for incumbent firms captures the effects of creative destruction on the stock market valuation of monopoly profits. In other words, during the diffusion of a GPT per capita consumption declines, the market interest rate rises, and the hazard rate increases. These changes lower the

³¹ Earlier contributions on this issue include the macroeconomic model of Cheng and Dinopoulos (1996) in which Schumpeterian waves obtain as a unique non-steady-state equilibrium solution and the current flow of monopoly profits follows a cyclical evolution.

per-capita expected discounted profits of the successful innovator and drives down its per capita stock market valuation.³² Furthermore, the larger the productivity gains associated with the new GPT, the larger the slump of the stock market. For example, it may be that the productivity gains generated by the introduction of the new GPT are large not because the new GPT is technologically very advanced at that initial stage, but because the previous GPTs are particularly inadequate for the needs of these sectors.³³ However, the size of the slump in the stock market is more severe, when the new GPT is diffused at a higher rate.

The aggregate stock value is given by the following equation:

$$\bar{V} = \left[\omega \frac{V_i}{N(t)} + (1 - \omega) \frac{V_g}{N(t)} \right] N(t), \quad (32)$$

where V_i and V_g are given by equation (20) after taking account equation (26). After substitution of these values into equation (32) and taking logs and derivatives with respect to time, I obtain the growth rate of the aggregate stock value:

³² Hobijn and Jovanovic (2001) argue that U.S. stock market decline in the early 1970s is due to the arrival of information technology and the fact that the stock-market incumbents were not ready to implement it. They state "Instead, new firms would bring in the new technology after the mid-1980s. Investors foresaw this in the early 1970s and stock prices fell right away." The U.S. stock market value relative to GDP plummeted to 0.4 in 1973, just after Intel had developed the microprocessor in late 1971. The decrease of the stock market value relative to GDP did not recover until the mid-1980s, and it then rose sharply. Leading OECD countries also experienced similar movements in their stock markets, following a U-shaped path.

³³ This was clearly the case for early computers, where even that valves had been getting smaller for over a decade prior to the arrival of the transistor, the transistor was still an order of magnitude smaller.

$$g_V = \frac{\dot{V}}{V} = g_N - \frac{(\mu - 1)\omega\delta(1 - \omega)}{[\omega + (1 - \omega)\mu]} \quad (33)$$

At the initial steady-state, where $\omega = 0$ and at the final steady-state, where $\omega = 1$, the growth rate of the aggregate stock value is equal to the rate of the population growth.

That is,

$$\dot{g}_V = g_N \quad (34)$$

The effects of a GPT on the stock market valuation of monopoly profits are summarized in the following proposition:

Proposition 4. *The growth rate of the stock market, g_V , depends negatively on the rate of GPT diffusion process, δ , and the magnitude of the GPT-ridden R&D productivity gains, μ , and positively on the rate of population growth, g_N . It also follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT.*

Proof. See Appendix B.

These comparative properties, which differentiate the model from several others in its class, can be illustrated with the help of Figure 3-7, which shows the growth rate of stock market as a function of the measure of industries that have adopted the new GPT. The initial adoption of the new GPT decreases the growth rate of the stock market below the rate of the population growth. In the later stages of the adoption of the new GPT, the growth rate of the stock market increases. When the diffusion process of the new GPT has been completed, the growth rate of the stock market is equal to the rate of the population growth. That is, it follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT.

This last result can be seen from the second term of the right hand side in equation (33).³⁴ The free entry condition in each R&D race (equation 20) implies that the per capita stock value in any industry $\theta (V_0/N(t))$ is constant over time. It jumps down instantaneously with the adoption of the new GPT, and it remains constant thereafter. The aggregate stock value, which increases exponentially at the rate of the population growth, jumps down with the arrival of the new GPT, and then increases again at the population growth rate. The slump in the aggregate stock value is due to the realization of the R&D productivity gain associated with the new GPT. The higher these R&D productivity gains are, the higher is the jump in the per capita industry and aggregate stock value at the time of the adoption of the new GPT.³⁵

An increase in the GPT diffusion rate, δ , increases the economywide resources devoted to R&D. Thus, the probability that the incumbent firm will be replaced by a follower firm increases. This can be seen from equation (18), which gives the value of monopoly profits. In other words, when the GPT diffusion process accelerates, the decrease in per capita consumption expenditure is more severe, and the per capita R&D investment increases. In this case, the U-shaped path of the growth rate of the stock market sags down (this is shown by the dotted-shaped curve in Figure 3-7). That is, the

³⁴ The numerator in equation (33), which is positive and it reflects the slope of a truncated S-curve, is equal to ϕ times a positive fraction that depends on the magnitude of the GPT-ridden R&D productivity gains and on the number of the industries that have adopted the new GPT.

³⁵ This can be seen from equation (33), where the first term in the right-hand side gets smaller when each industry adopts the new GPT relative to the second term of the right-hand side of the same equation.

slope of the growth of the stock market gets steeper at the initial stages of the diffusion process of the new GPT and it gets flatter at the final stages of this process.³⁶

An increase in the productivity gains generated by the new GPT, μ , lowers the cost of discovering the next higher quality product. This, in turn, will affect negatively the stock market valuation of the incumbent firm (see equation (20)). An increase in the rate of population growth, g_N , shifts the U-shaped curve in Figure 3-7 upwards and increases the growth rate of the stock market.

3.4.2 Aggregate Investment

Proposition 5. *The effect of the GPT diffusion on the aggregate investment during the adoption process is ambiguous.*

Proof. See Appendix B.

The initial steady-state equilibrium is at point A in Figure 3-5. There is an upward jump in the aggregate investment with the introduction of the new GPT (from $\hat{I}$ to $\tilde{I}$). Along the diffusion path, both innovation rates (I_0 and I_1) decrease until the economy reaches at the final steady-state equilibrium point B, where the aggregate investment is higher relative to the initial steady-state equilibrium point A. There is an upward jump on

³⁶ The first OPEC shock may also explain a part of the drop in the stock market in the early 1970s, as well as a part of the productivity slowdown. Hobijn and Jovanovic (2001) argue that there are several problems associated with the oil-shock explanation. One problem is that a rise in oil prices should have lowered current profits more than future profits, because of the greater ease of finding substitutes for oil on the long-run, perhaps current output more than future output and, therefore, should have produced a rise in the ratio of market capitalization to GDP, not a fall. This scenario also implies a constant entry in the stock market, something that contradicts their evidence. Another problem that is associated with the oil-price-shock explanation for the stock-market drop is that the energy-intensive sectors did not experience the largest drop in value in 1973-1974. Their evidence supports that the information-technology-intensive sectors experienced the largest drop in 1973-1974.

the innovation rate of the industries that have not adopted the new GPT at the final steady state (from point B_0 to B). One possible picture of how the aggregate investment behaves along the diffusion of the new GPT is shown in Figure 3-5. Along the transition path, the aggregate investment decreases and then increases. In the initial stages of the diffusion process, only a limited number of industries adopt the new GPT (see equation (1)). These industries are called the early adopters. As more industries adopt the new GPT, the aggregate investment increases.

3.5 Concluding Remarks

Previous models that have analyzed GPTs exhibit the scale effects property. The present paper analyzed the effects of a GPT on short-run and long-run Schumpeterian growth without scale effects. The absence of growth scale effects and the modeling of the diffusion process through *S*-curve dynamics generate several novel and interesting results.

First, the long-run growth rate of the economy depends positively on the magnitude of quality innovations. Any policy that affects this magnitude has long-run growth effects. However, the absence of the arrival of a new GPT in the economy does not reduce the long-run growth rate to zero as in the previous GPTs-based growth models. All the previous R&D-based models that analyze the effects of GPTs exhibit scale effects.

The assumption that the diffusion of the new GPT follows an *S*-curve generates two steady-state equilibria: one is the initial steady-state before the adoption of the new GPT begins and the other is the final steady-state after the diffusion process of the new GPT has been completed. At the final steady-state relative to the initial steady-state: the long-run growth rate is higher, the aggregate investment is higher, the per-capita

consumption expenditure is lower, and the market interest rate is equal to the subjective discount rate.

The growth rate of the stock market depends negatively on the rate of GPT diffusion process, and the magnitude of the GPT-ridden R&D productivity gains, and positively on the rate of population growth. It also follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT. This is consistent with the empirical evidence provided by Jovanovic and Rousseau (2001) who empirically document that during times of rapid technological change the growth of the stock market slows down, since the new entrants of the stock market will grab the most value from previous entrants because the incumbents will find hard to keep up.

One could also develop a dynamic general equilibrium model to study the effects of a GPT diffusion on a global economy that exhibits endogenous Schumpeterian growth. As in this model, the adoption of a GPT by a particular industry can generate an increase in the productivity of R&D workers and the magnitude of all future innovations and its diffusion across industries can be governed by *S*-curve dynamics. The diffusion of the GPT within an industry from one country to the other can occur with a time lag. Under this framework, it would be interesting to analyze the long-run and transitional dynamic effects of a new GPT on trade patterns, product cycles and (transitional) divergence in per-capita growth rates between the two countries. This is a fruitful direction for future research.

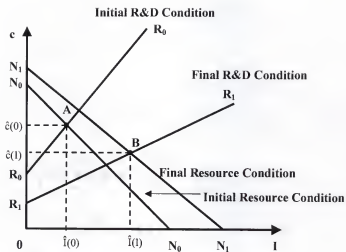


Figure 3-1. Steady-State Equilibria:

Point A: No industry has adopted the new GPT.

Point B: All industries have adopted the new GPT.

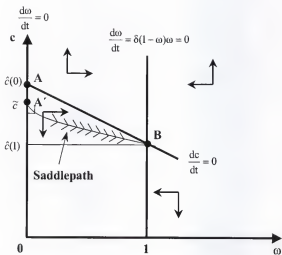


Figure 3-2. Stability of the Balanced-Growth Equilibrium

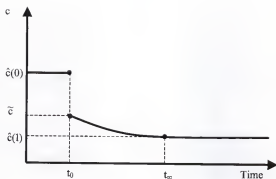


Figure 3-3. Time Path of the Per Capita Consumption Expenditure After a GPT Arrives in the Economy

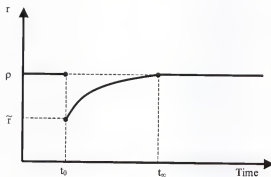


Figure 3-4. Time Path of the Market Interest Rate After a GPT Arrives in the Economy

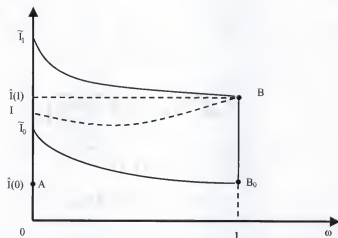


Figure 3-5. Evolution of the Aggregate Investment During the Diffusion Path:

The initial steady-state equilibrium is at point A. There is an upward jump in the aggregate investment with the introduction of the new GPT. One possible path of the aggregate investment, along the diffusion path, is depicted in the Figure by the dotted curve IB.

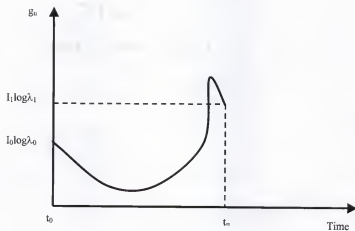


Figure 3-6. Effects of a GPT on the Schumpeterian Growth Rate:
When all industries have adopted the new GPT, the economy experiences higher steady-state Schumpeterian growth. There also exist transitional growth cycles of per capita GNP.

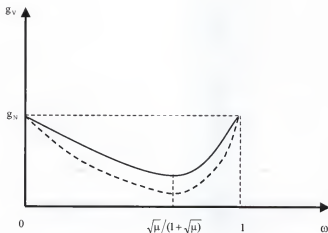


Figure 3-7. Effects of a GPT on the Stock Market:

The growth rate of the stock market depends on the rate of GPT diffusion process, the magnitude of the GPT-ridden R&D productivity gains, and the rate of population growth. It also follows a U-shaped path relative to the population growth.

CHAPTER 4

SUSTAINED COMPARATIVE ADVANTAGE IN A MODEL OF SCHUMPETERIAN GROWTH WITHOUT SCALE EFFECTS

4.1 Introduction

Many models of endogenous growth and trade emphasize the role of continual product innovation based on R&D investment in determining the pattern of trade between countries. Grossman and Helpman (1991*a, b, c*) have developed models where innovations lead to either improvements in the quality of existing products ("quality ladders" models) or increase in the variety of the goods ("love for variety" models). Taylor (1993) has extended the continuum Ricardian model of Dornbusch et al. (1977) based on the "quality ladders" approach by Grossman and Helpman. All these studies exhibit the scale effect property: if one incorporates population growth in these models, then the size of the economy (scale) increases exponentially over time, R&D resources grow exponentially, and so does the long-run growth rate of per-capita real output. The scale effects property is a consequence of the assumption that the growth rate of knowledge is directly proportional to the level of resources devoted to R&D. Jones (1995*a*) has argued that the scale effects property of earlier endogenous growth models is inconsistent with post-war time series evidence from all major advanced countries that shows an exponential increase in R&D resources and a more-or-less constant rate of per-capita GDP growth. Jones's criticism has stimulated the development of a new class of models that generate growth without scale effects.³⁷ However, the theoretical literature on

³⁷ See Dinopoulos and Thompson (1999) for more details on this issue.

trade and growth without scale effects has focused either on closed economy models or on structurally identical economies engaging in trade with each other.³⁸ This chapter develops a two-country general equilibrium framework without scale effects to determine the equilibrium relative wages and the pattern of trade between countries.

My approach borrows from Taylor's work (1993) in that industries differ in production technologies. In his model, industries also differ in research technologies and in the set of technological opportunities available for each industry. In the presence of heterogeneous research technologies (captured by different productivity in R&D services), the pattern of R&D production and the pattern of goods production within each country can differ. As a result, there is a case for trade between countries in R&D services. The absence of heterogeneity in research technologies in my model makes the removal of scale effects more tractable, but eliminates the need for trade in R&D services between countries.

In the present model, there are two countries that may differ in relative size: Home and Foreign. The population in each country grows at a common positive and exogenously given rate and labor is the only factor of production. There is a continuum of industries producing final consumption goods. Labor in each industry can be allocated between the two economic activities, manufacturing of high-quality goods and R&D services, which are used to discover new products of higher quality. As in Grossman and Helpman's (1991c) version of the quality-ladders growth model, the quality of each final

³⁸ Dinopoulos and Syropoulos (2001) have recently developed a two-country general equilibrium model of endogenous Schumpeterian (R&D based) growth without scale effects to examine the effect of globalization on economic growth when countries differ in population size and relative factor endowments.

good can be improved through endogenous innovation. The arrival of innovations in each industry is governed by a memoryless Poisson process whose intensity depends positively on R&D investments and negatively on the rate of difficulty of conducting R&D. I consider two alternative specifications regarding the difficulty of conducting R&D in order to remove the scale effects property. The first specification is called the permanent effects of growth (PEG) and it has been proposed by Dinopoulos and Thompson (1996). According to this specification, R&D becomes more difficult over time and the degree of R&D difficulty is proportional to the size of the world market.

The second specification is called the temporary effects of growth (TEG) and it has been proposed by Segerstrom (1998). With this specification, R&D also becomes more difficult over time but the degree of difficulty is an increasing function of cumulative R&D effort in each industry.

The removal of scale effects property under either specification regarding the R&D difficulty leaves some predictions in Taylor's (1993) model robust. A uniform increase in Home's unit production costs reduces its relative wage and reduces its range of goods that Home produces. A uniform increase in Foreign's unit production costs increases Home's relative wage and increases Home's comparative advantage. In addition, Home's relative wage and its comparative advantage are left unaffected by equiproportionate changes in Home and Foreign production costs. Finally, factor price equalization is not a generic property of the steady-state equilibrium. However, several comparative-steady-state results in Taylor's (1993) model change with the removal of the scale effects property. First, in his model, the direction of the effect of the size of innovations (which can vary across industries) on the pattern of goods production, R&D

production, the pattern of trade, and the relative wage depends on the assumption that the size of innovations is heterogeneous. Under the heterogeneity assumption, the increase in the inventive step creates a deficit in the balance of payments for Home because it raises the royalties' payments that Home has to pay for using the front-line technology.³⁹

Balance of payments is maintained through two adjustments; Home raises its goods trade balance by increasing the range of goods produced at Home and it reduces its reliance on imported R&D by conducting more itself. Removing part of this heterogeneity in his model, by eliminating Home's relative advantage in goods versus R&D, results in zero trade in R&D and no effect of the size of innovations on the pattern of trade and Home's relative wage.⁴⁰ On contrast, in the present model, the direction of the effect of the size of innovations on the pattern of trade and Home's relative wage depends on the way in which the scale effects property is removed. Under the PEG specification, changes in the size of innovations do not affect Home's comparative advantage and its relative wage, while under the TEG specification, an increase in the size of innovations reduces Home's relative wage and raises its comparative advantage.

Under the PEG specification, the increase in the size of innovations increases the profitability of R&D activity. As a result, more firms enter into R&D races until the

³⁹ Taylor (1993) divides the world's available technologies into two sets: the set of front line technologies and the set of backward technologies. Frontline technologies are those that are minimum cost given the prevailing wage rate. He further assumes that when an innovator located in Foreign succeeds in the global R&D races and discovers the front line technology, it has two options: it can either implement this improvement on the foreign technology or it can go multinational and carry the innovation abroad to a wholly owned subsidiary. This subsidiary would then pay the foreign firm a royalty.

⁴⁰ Eliminating the across country heterogeneity in his model, results in factor price equalization and indeterminate pattern of trade in both goods and R&D.

expected profits are driven back to zero. The adjustment to equilibrium requires an increase in the level of global R&D investment, while it leaves Home's relative wage and its comparative advantage unchanged. On contrast, the TEG specification implies that the global level of R&D investment depends positively on the rate of population growth and negatively on the R&D difficulty parameter, since the R&D difficulty grows at the constant rate of population growth rate at the steady-state equilibrium. Therefore, an increase in the size of innovations raises the rewards of innovating and thus it induces more firms to engage in R&D. However, since the long-run level of R&D activity is fixed by the unchanged parameters of the model, to maintain the equilibrium, the relative wage has to decrease. The decrease in Home's relative wage raises its comparative advantage.

Second, the direction of the effect of a change in R&D costs in Taylor's (1993) model depends on the heterogeneous research assumption. A uniform increase in Home's R&D costs reduces Home's relative wage and raises the measure of Home industries active in production. A uniform increase in Foreign's costs increases Home's relative wage and reduces the measure of Home industries active in production. However, if one considers an equiproportionate changes in R&D costs in his model, then the effect of these changes on Home's relative wage and its comparative advantage is ambiguous.⁴¹

⁴¹ This ambiguity arises because R&D technologies play two roles in Taylor's (1993) model. First, R&D technologies determine the competitive margin for R&D production within each country and the division of R&D across countries. Second, R&D technologies determine, via the free entry process into the R&D, the value of a patent in any industry. Equiproportionate changes in Home and Foreign unit R&D labor requirements increase the value of patents and thus raise world expenditure levels. The effect of the increased world expenditure on Home produced goods is ambiguous without making further assumptions.

Eliminating the heterogeneous research technologies assumption, results in factor price equalization in any equilibrium with both countries conducting R&D. In this case, an equiproportionate changes in R&D costs in his model, has no effect on Home's relative wage and its comparative advantage. Furthermore, if one assumes the absence of heterogeneous size of innovations and unity budget share for products in Taylor's model, then an equiproportionate changes in R&D costs makes Home conduct less R&D.

On contrast, in the present model, the direction of the effect of an equiproportionate change of the unit R&D labor requirement depends on the way in which the scale effects property is removed. Under the PEG specification, an increase in R&D costs reduces Home's relative wage and raises its comparative advantage, while under the TEG specification, changes in R&D costs do not affect Home's relative wage and its comparative advantage.

Under the PEG specification, the increase in R&D costs decreases the profitability of R&D activity. As a result, fewer firms enter into R&D races until the expected profits are driven back to zero. The adjustment to equilibrium requires a decrease in the level of Home's relative wage. The decrease in Home's relative wage raises its comparative advantage. On contrast, under the TEG specification, since the long-run level of R&D activity is fixed by the unchanged parameters of the model, an increase in R&D costs, increases the population-adjusted difficulty of R&D and thus leaving Home's relative wage and its comparative advantage unchanged.

The analysis in the present model generates new additional findings. Under both specifications, the model generates a unique steady-state equilibrium in which there is complete specialization in both goods and R&D production within each industry. Trade

between the two countries occurs only in goods and not in R&D services. In contrast to the work of Grossman and Helpman (1991c), factor price equalization does not hold in the steady-state equilibrium under either specification (Propositions 1 and 5).

Under the PEG specification, Home's relative wage depends positively on the Foreign's relative size, Foreign's unit labor requirement in manufacturing, and the population growth rate. It depends negatively on the R&D difficulty parameter, the unit R&D labor requirement, the consumer's subjective discount rate, and Home's unit labor requirement in manufacturing (Proposition 2). In contrast to previous models (Grossman and Helpman (1991c), and Taylor (1993)), these results highlight the effects of population growth and the R&D difficulty on relative wages.

Under the PEG specification, the range of goods produced in Home and exported depends positively on the R&D difficulty parameter, the unit R&D labor requirement, the consumer's subjective discount rate, and Foreign's unit labor requirement in manufacturing. It depends negatively on Foreign's relative size, the population growth rate, and Home's unit labor requirement in manufacturing (Proposition 3). These results also highlight the importance of population growth and the R&D difficulty parameter on the pattern of goods and R&D production, and the pattern of trade between the two countries.

In addition, when the scale effects property is removed via the PEG specification, the global innovation rate depends positively on Home's relative size measured by its share of world's population, Foreign's relative size measured by its share of world's population, and the population growth rate. It depends negatively on the R&D difficulty

parameter, the consumer's subjective discount rate, and the unit R&D labor requirement (Proposition 4).

Some results that hold under the PEG specification analysis are reversed when the TEG specification is introduced in the model. Home's relative wage depends positively on the consumer's subjective discount rate and the R&D difficulty growth parameter. It depends negatively on the population growth rate (Proposition 6). The range of goods Home produces and exports depends positively on the population growth rate. It depends negatively on the consumer's subjective discount rate and the R&D difficulty growth parameter (Proposition 7).

The global level of R&D investment, under the TEG specification is completely determined by the exogenous rate of population growth and the R&D difficulty growth parameter. Specifically, the global innovation rate is higher when the population of consumers grows faster or when R&D difficulty increases more slowly over time (Proposition 8).

This chapter is organized as follows. Section 4.2 outlines the features of the model. Section 4.3 describes the steady state equilibrium of the model under the PEG specification and section 4.4 presents the comparative steady state results under the PEG specification. Section 4.5 analyzes the steady state equilibrium of the model under the TEG specification and section 4.6 presents the comparative steady state results under the TEG specification. Section 4.7 concludes this chapter by summarizing the key findings and suggesting possible extensions. The algebraic details and proofs of propositions in this chapter are relegated to Appendix C.

4.2 The Model

This section develops a two-country, dynamic, general-equilibrium model with the following features. Each country engages in two activities: the production of final consumption goods and research and development. Each of the two economies is populated by a continuum of industries indexed by $\theta \in [0, 1]$. A single primary factor, labor, is used in both goods and R&D production for any industry. In each industry θ firms are distinguished by the quality j of the products they produce. Higher values of j denote higher quality and j is restricted to taking on integer values. At time $t=0$, the state-of-the-art quality product in each industry is $j=0$, that is, some firm in each industry knows how to produce a $j=0$ quality product and no firm knows how to produce any higher quality product. The firm that knows how to produce the state-of-the-art quality product in each industry is the global leader for that particular industry. At the same time, challengers in both countries engage in R&D to discover the next higher-quality product that would replace the global leader in each industry. If the state-of-the-art quality in an industry is j , then the next winner of an R&D race becomes the sole global producer of a $j+1$ quality product. Thus, over time, products improve as innovations such as push each industry up its "quality ladder," as in Grossman and Helpman (1991c).

I assume for simplicity, that all firms in the global economy know how to produce all products that are at least one step below the state-of-the-art quality product in each industry. This assumption, which is standard in most quality-ladders growth models, prevents the incumbent monopolist from engaging in further R&D, which is standard assumption in most quality-ladder models.

For clarity, I adopt the following conventions regarding notation. Henceforth, superscripts "h" and "f" identify functions and variables of "Home" and "Foreign" countries, respectively. Functions and variables without superscripts are related to the global economy, while functions and variables with subscripts are related to activities and firms within an industry.

4.2.1 Household Behavior

Let $N^i(t)$ be country i 's population at time t . I assume that each country's population is growing at a common constant, exogenously given rate $g_N = \dot{N}^i(t)/N^i(t) > 0$. In each country there is a continuum of identical dynastic families that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. I normalize the measure of families in each country at time 0 to equal unity. Thus, the population of workers at time t in country i is $N(t)^i = e^{g_N t}$.

Each household in country i maximizes the discounted utility⁴²

$$U = \int_0^\infty e^{-(\rho - g_N)t} \log u(t) dt, \quad (1)$$

where $\rho > 0$ is the constant subjective discount rate. In order for U to be bounded, I assume that the effective discount rate is positive (i.e., $\rho - g_N > 0$). Expression $\log u(t)$ captures the per capita utility at time t , which is defined as follows:

$$\log u(t) = \int_0^1 \log \left[\sum_j \lambda^j q(j, \theta, t) \right] d\theta. \quad (2)$$

⁴² Barro and Sala-i-Martin (1995, Ch. 2) provide more details on this formulation of the household's behavior within the context of the Ramsey model of growth.

In equation (2), $q(j, \theta, t)$ denotes the quantity consumed of a final product of quality j (i.e., the product that has experienced j quality improvements) in industry $\theta \in [0, 1]$ at time t . Parameter $\lambda > 1$ measures the size of quality improvements (i.e., the size of innovations).

At each point in time t , each household allocates its income to maximize (2) given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function for the product in each industry with the lowest quality-adjusted price

$$q'(j, \theta, t) = \frac{c'(t)N'(t)}{p'(t)}, \quad (3)$$

where $c'(t)$ is country i 's per capita consumption expenditure, and $p(t)$ is the market price of the good considered. Because goods within each industry adjusted for quality are by assumption identical, only the good with the lowest quality-adjusted price in each industry is consumed. The quantity demanded of all other goods is zero. The global demand for a particular product is given by aggregating equation (3) across the two countries to obtain

$$q(j, \theta, t) = \sum_{i=h,f} q'(j, \theta, t). \quad (4)$$

Given this static demand behavior, the intertemporal maximization problem of country i 's representative household is equivalent to

$$\max_{c'(t)} \int_0^{\infty} e^{-(\rho - z_N)t} \log c'(t) dt, \quad (5)$$

subject to the intertemporal budget constraint $\dot{a}'(t) = r'(t)a(t) + w'(t) - c'(t) - g_N a'$,

where $a'(t)$ denotes the per capita financial assets in country i , $w'(t)$ is the wage income of the representative household member in country i , and $r'(t)$ is country i 's

instantaneous rate of return at time t . The solution to this maximization problem obeys the well-known differential equation

$$\frac{\dot{c}'(t)}{c'(t)} = r'(t) - \rho, \quad (6)$$

Equation (6) implies that a constant per-capita consumption expenditure is optimal when the instantaneous interest rate in each country equals the consumer's subjective discount rate ρ .

4.2.2 Product Markets

In each country firms can hire labor to produce any final consumption good $\theta \in [0, 1]$. Let $L^i(\theta, t)$ and $Q^i(\theta, t)$ respectively denote the amounts of labor devoted in manufacturing of final consumption good θ in country i and the output of final consumption good θ in country i . Then the production function of the final consumption good θ in country i is given by the following equation

$$Q^i(\theta, t) = \frac{L^i(\theta, t)}{\alpha_Q^i(\theta)}, \quad (7)$$

where $\alpha_Q^i(\theta)$ is the unit labor requirement associated with the final consumption good θ in country i . I assume that each vertically differentiated good must be manufactured in the country in which the most recent product improvement has taken place. That is, I rule out international licensing and multinational corporations.⁴³

⁴³ Taylor (1993) incorporates multinational corporations in a model of endogenous growth and trade. In his model, innovations are always implemented on front line production technologies (i.e., technologies that are minimum cost given the prevailing wage rates) and when innovation and implementation occur at different countries, the resulting transactions are considered as imports and exports of R&D.

Following Dornbusch et al. (1977), the relative labor unit requirement for each good θ is given by

$$A(\theta) = \frac{\alpha_Q^f(\theta)}{\alpha_Q^h(\theta)}, \quad A'(\theta) < 0. \quad (\text{A.1})$$

The relative unit labor requirement function in (A.1) is by assumption continuous, and decreasing in θ .

The assumptions that goods within an industry are identical when adjusted for quality and Bertrand price competition in product markets imply that the monopolist in each industry engages in limit pricing. The assumption that the technology of all inferior quality products is public knowledge imply that the quality leader charges a single price, which is λ times the lowest manufacturing cost between the two countries:

$$p = \lambda \cdot \min \{ \alpha_Q^h(\theta) w^h, \alpha_Q^f(\theta) w^f \}. \quad (8)$$

I choose the wage of foreign labor, w^f , as the numeraire of the model by setting:

$$w^f = 1. \quad (9)$$

I also assume that the wage of home labor, w^h , which is also Home's relative wage, ω , is greater than one⁴⁴

$$w^h = \omega > 1. \quad (10)$$

Assumption (10) implies that the price of every top quality good is equal to

$$p = \lambda \alpha_Q^f(\theta). \quad (11)$$

It follows that the stream of profits of the incumbent monopolist that produces the state-of-the-art quality product in Home will be equal to

⁴⁴ In Propositions 1 and 5, I provide sufficient conditions under which this assumption holds.

$$\pi^h(\theta, t) = [\lambda \alpha_Q^h(\theta) - a_Q^h(\theta) \omega] q = \left(1 - \frac{\omega \alpha_Q^h(\theta)}{\lambda \alpha_Q^h(\theta)} \right) E(t), \quad (12)$$

while the stream of profits of the incumbent monopolist that produces the state-of-the-art quality product in Foreign will be equal to

$$\pi^f(\theta, t) = [\lambda \alpha_Q^f(\theta) - a_Q^f(\theta)] q = \frac{(\lambda - 1)}{\lambda} E(t), \quad (13)$$

where $E(t) = [c^h(t)N^h(t) + c^f(t)N^f(t)]$ is the world expenditure on final consumption goods.

4.2.3 R&D Races

Labor is the only input engaged in R&D in any industry $\theta \in [0, 1]$. Let $L'_R(\theta, t)$ and $R'(\theta, t)$ respectively denote the amounts of labor devoted in R&D services in industry θ in country i and the output of R&D services in industry θ in country i . The production function of R&D services in industry θ in country i exhibits constant returns and is given by the following equation⁴⁵

$$R'(\theta, t) = \frac{L'_R(\theta, t)}{\alpha_R}, \quad (14)$$

where α_R is the unit labor requirement in the production of R&D services. Note that the absence of a superscript and the absence of the industry index θ in the unit labor requirement imply that they are the same across countries, industries and goods of

⁴⁵ The empirical evidence on returns to scale of R&D expenditure is inconclusive. Segerstrom and Zolnerek (1999) among others developed a model where they allow for diminishing returns to R&D effort at the firm level and industry leaders have R&D cost advantages over follower firms. In their model, when there are diminishing returns to R&D and the government does not intervene both industry leaders and follower firms invest in R&D.

different quality levels. The absence of heterogeneous research technologies allows me to focus on the implications of assumption (A.1) on the properties of the model.⁴⁶

In each industry θ there are global, sequential and stochastic R&D races that result in the discovery of higher-quality final products. A challenger firm k that is located in country $i \in \{h, f\}$ targeting a quality leader in country $i \in \{h, f\}$ engages in R&D in industry θ and discovers the next higher-quality product with instantaneous probability $I_k^i(\theta, t)dt$, where dt is an infinitesimal interval of time and

$$I_k^i(\theta, t) = \frac{R_k^i(\theta, t)}{X(t)}, \quad (15)$$

where $R_k^i(\theta, t)$ denotes firm k 's R&D outlays and $X(t)$ captures the difficulty of R&D in industry θ at time t . I assume that the returns to R&D investments are independently distributed across challengers, countries, industries, and over time. Therefore, the industry-wide probability of innovation can be obtained from equation (14) by summing up the levels of R&D across all challengers in that country. That is,

$$I^i(\theta, t) = \sum_k I_k^i(\theta, t) = \frac{R^i(\theta, t)}{X(t)}, \quad (16)$$

where $R^i(\theta, t)$ denotes total R&D services in industry θ in country i . Variable $I^i(\theta, t)$ is the effective R&D.⁴⁷ The arrival of innovations in each industry follows a memoryless

⁴⁶ Taylor (1993) has introduced heterogeneity in the research technologies and in the technological opportunity for improvements in technologies. The presence of heterogeneous research technologies makes trade in R&D services between countries possible. The absence of heterogeneous research technologies in the present model, makes the removal of scale effects more tractable, but eliminates the possibility of trade in R&D services between the two countries.

⁴⁷ The variable $I^i(\theta, t)$ is the intensity of the Poisson process that governs the arrivals of innovations in industry θ in country i . Dinopoulos and Syropoulos (2001) model the

Poisson process with intensity $I(\theta, t) = \sum_i R^i(\theta, t)/X(t)$ which equals the global rate of innovation in a typical industry. The function $X(t)$ has been introduced in the endogenous growth literature after Jone's (1995a) empirical criticism of R&D based growth models generating scale effects.

A recent body of theoretical literature has developed models of Schumpeterian growth without scale effects.⁴⁸ Two alternative specifications have offered possible solutions to the scale-effects property. The first specification proposed by Dinopoulos and Thompson (1996) removes the scale-effects property by assuming that the level of R&D difficulty is proportional to the market size measured by the level of population, $X(t) = kN(t)$,

(17)

where $k > 0$ is a parameter.

This specification captures the notion that it is more difficult to introduce new products and replace old ones in a larger market. The model that results from this specification is called the permanent effects of growth (PEG) model because policies such as an R&D subsidy and tariffs can alter the per-capita long-run growth rate.⁴⁹

The second specification proposed by Segerstrom (1998) removes the scale

strategic interactions between a typical incumbent and its challengers as a differential game for Poisson jump processes and derive the equilibrium conditions that govern the solution to a typical R&D contest. They also provide an informal and intuitive derivation of these conditions. In the present model, I follow their informal derivation to derive my results.

⁴⁸ See Dinopoulos and Thompson (1999) for an overview of these models.

⁴⁹ Dinopoulos and Thompson (1998) provide micro foundations for this specification in the context of a model with horizontal and vertical product differentiation.

effects property by assuming that R&D becomes more difficult over time because "the most obvious ideas are discovered first." The model that results from this specification is called the temporary effects of growth (TEG) model. In this model, the long-run growth rate is proportional to the exogenous rate of population growth and it is not affected by any standard policy instruments.

Under the TEG specification, R&D starts being equally difficult in all industries ($X(\theta, 0) = 1$ for all θ), and the level of R&D difficulty grows according to

$$\frac{\dot{X}(t)}{X(t)} = \mu[I^h(\theta, t) + I^f(\theta, t)] = \mu I(\theta, t), \quad (18)$$

where $\mu > 0$ is a constant.

In subsequent sections I will consider each specification separately to analyze the steady-state equilibria and to derive their comparative-static properties.

Consider now the stock-market valuation of temporary monopoly profits.

Consumer savings are channeled to firms engaging in R&D through the stock market.

The assumption of a continuum of industries allows consumers to diversify the industry-specific risk completely and earn the market interest rate. At each instant in time, each challenger issues a flow of securities that promise to pay the flow of monopoly profits if the firm wins the R&D race and zero otherwise.⁵⁰ Consider now the stock-market valuation of the incumbent firm in each industry. Let $V^i(t)$ denote the expected global discounted profits of a successful innovator at time t in country i , when the global monopolist charges a price p for the state-of-the-art quality product. Because each global

⁵⁰ If the monopolist is located in Home, the monopoly profits are defined by equation (12) and if the monopolist is located in Foreign the monopoly profits are defined by equation (13).

quality leader is targeted by challengers from both countries who engage in R&D to discover the next higher-quality product, a shareholder faces a capital loss $V^i(t)$ if further innovation occurs. The event that the next innovation will arrive occurs with instantaneous probability $I dt$, whereas the event that no innovation will arrive occurs with instantaneous probability $1 - I dt$. Over a time interval dt , the shareholder of an incumbent's stock receives a dividend $\pi(t)dt$ and the value of the incumbent appreciates by $dV^i(t) = [\partial V^i(t)/\partial t]dt = \dot{V}^i(t)dt$. Perfect international capital mobility implies that $r^h = r^f = r$. The absence of profitable arbitrage opportunities requires the expected rate of return on stock issued by a successful innovator to be equal to the riskless rate of return r ; that is,

$$\frac{\dot{V}^i(\theta, t)}{V^i(\theta, t)}[1 - I(\theta, t)dt]dt + \frac{\pi^i(\theta, t)}{V^i(\theta, t)}dt - \frac{[V^i(\theta, t) - 0]}{V^i(\theta, t)}I(\theta, t)dt = rdt. \quad (19)$$

Taking limits in equation (19) as $dt \rightarrow 0$ and rearranging terms appropriately gives the following expression for the value of monopoly profits

$$V^i(\theta, t) = \frac{\pi^i(\theta, t)}{r(t) + I(\theta, t) - \frac{\dot{V}^i(\theta, t)}{V^i(\theta, t)}}. \quad (20)$$

A typical challenger k located in country i chooses the level of R&D investment $R_k^i(\theta, t)$ to maximize the following expected discounted profits:

$$V^i(\theta, t) \frac{R_k^i(\theta, t)}{X(t)} dt - w^i \alpha_R R_k^i(\theta, t) dt, \quad (21)$$

where $I_k^i dt = [R_k^i(\theta, t)/X(t)]dt$ is the instantaneous probability of discovering the next higher-quality product and $w^i \alpha_R R_k^i(\theta, t)$ is the R&D cost of challenger k located in country i .

Free entry into each R&D race drives the expected discounted profits of each challenger down to zero and yields the following zero profit condition:

$$V'(t) = w^t \alpha_R X(t). \quad (22)$$

The pattern of R&D production across the two countries can be determined by utilizing equations (20) and (22). Evaluating these equations on the competitive margin in R&D production, $\tilde{\theta}$, I can obtain the R&D schedule (i.e., the schedule of relative labor productivities in goods) as follows

$$\omega = RD(\tilde{\theta}) = \frac{\alpha_Q^f(\tilde{\theta})}{\alpha_Q^h(\tilde{\theta})}, \quad (23)$$

where $RD(\tilde{\theta})$ is continuous and decreasing in $\tilde{\theta}$. For low values of θ , Home has higher relative labor productivity than Foreign, and thus it earns higher wage. Therefore, Home has comparative advantage in producing and conducting R&D the final goods with lower θ and Foreign has comparative advantage in producing and conducting R&D the final goods with higher θ . The R&D schedule can be depicted in Figure 4-1.

Lemma 1. *Under assumption (A.1) and for any given value of the relative wage, $\omega \in (\alpha_Q^f(1)/\alpha_Q^h(1), \alpha_Q^f(0)/\alpha_Q^h(0))$, there exists an industry $\tilde{\theta}$ defined by equation (23) such that*

- (a) *firms are indifferent between conducting R&D in Foreign or in Home,*
- (b) *for each industry $\theta \in [0, \tilde{\theta})$, only Home conducts R&D,*
- (c) *for each industry $\theta \in (\tilde{\theta}, 1]$, only Foreign conducts R&D.*

Proof. See Appendix C.

One can find the results from Lemma 1 in Dornbusch et al. (1977). However, the derivation of Lemma 1 differs between the present model and the one in Dornbusch et al. (1977). In their model, the results from Lemma 1 come from the assumption of perfect competition in all markets. In the present model, the intuition behind Lemma 1 results from the zero profit conditions regarding R&D. If in industry θ , R&D is undertaken by Home, then the zero profit conditions for R&D imply that Foreign has negative profits in this particular industry (see equations (20) and (22)). Thus, for all industries that Home undertakes R&D, Foreign has negative profits and does not engage in R&D in these industries. The reverse is true for those industries that Foreign undertakes R&D. Home has negative profits in these industries, so it does not engage in R&D in those industries. Thus, both countries sustain their comparative advantage.

4.2.4 Labor Markets

Consider first the Home labor market. All workers are employed by firms in either production or R&D activities. Taking into account that each industry leader charges the same price p and that consumers only buy goods from industry leaders in equilibrium, it follows from (7) that total employment of labor in production in Home is $\int_0^{\bar{\theta}} Q^h(\theta, t) \alpha_Q^h d\theta$. Solving equation (14) for each industry leader's R&D employment $L_R^h(\theta, t)$ and then integrating across industries, total R&D employment by industry leaders in Home is $\int_0^{\bar{\theta}} R^h(\theta, t) \alpha_R^h d\theta$. Thus, the full employment of labor condition for Home at time t is given by

$$N^h(t) = \int_0^{\bar{\theta}} Q^h(\theta, t) \alpha_Q^h(\theta) d\theta + \int_0^{\bar{\theta}} R^h(\theta, t) \alpha_R^h d\theta. \quad (24)$$

I can derive in a similar way the full employment of labor condition for Foreign at time t and obtain

$$N^f(t) = \int_0^1 Q^f(\theta, t) \alpha_Q^f(\theta) d\theta + \int_{\tilde{\theta}}^1 R^f(\theta, t) \alpha_R d\theta. \quad (25)$$

Equations (24) and (25) complete the description of the model.

4.3 Steady-State Equilibrium Under the PEG Specification

In this section I derive the steady-state equilibrium under the PEG specification proposed by Dinopoulos and Thompson (1996), which is described according to equation (17). Assuming that the relative wage, ω , is constant over time at the steady-state equilibrium, equation (22) implies that $V'(\theta, t)/V'(\theta, t) = \dot{X}(t)/X(t) = g_N$. That is, the expected global discounted profits of a successful innovator at time t in country i , $V^i(t)$, and the level of R&D difficulty, $X(t)$, grow at the constant rate of population growth, g_N . In the steady-state equilibrium, the market interest rate, r , must be equal to the subjective discount rate, ρ .⁵¹ Combining equations (20) and (22) after taking into account equation (17), I obtain the following zero profit conditions for Home and Foreign respectively for each industry:

$$\frac{\left(1 - \frac{\omega \alpha_Q^b(\theta)}{\lambda \alpha_Q^f(\theta)}\right) E(t)}{\rho + I(\theta, t) - g_N} = \omega \alpha_R k N(t), \quad \forall \theta \in [0, \tilde{\theta}) \quad (26)$$

$$\frac{\frac{(\lambda - 1)}{\lambda} E(t)}{\rho + I(\theta, t) - g_N} = \alpha_R k N(t), \quad \forall \theta \in (\tilde{\theta}, 1]. \quad (27)$$

⁵¹ This property is shared by all Schumpeterian models where growth is generated by the introduction of final consumption (as opposed to intermediate production) goods.

Utilizing equations (26) and (27), one can rank the level of global R&D investment between Home and Foreign. The ranking of the level of global R&D investment between Home and Foreign is illustrated graphically in Figure 4-2. Notice that the level of global R&D investment, I , does not depend on θ for $\theta \in (\tilde{\theta}, 1]$. On contrast, the level of global R&D investment, I , depends on θ for $\theta \in [0, \tilde{\theta})$. As Home conducts R&D in more industries, the level of global R&D investment increases.⁵²

Integrating equation (26) over $[0, \tilde{\theta})$ and equation (27) over $(\tilde{\theta}, 1]$, I obtain the following zero profit conditions for home and foreign, respectively at the economy-wide level:

$$\left(\tilde{\theta} - \frac{\omega}{\lambda} \int_0^{\tilde{\theta}} \frac{\alpha_Q^h(\theta)}{\alpha_Q^f(\theta)} d\theta \right) \bar{E}(t) = \tilde{\theta} \omega \alpha_R k [\rho + I(\theta, t) - g_N], \quad (28)$$

$$\frac{(\lambda - 1)}{\lambda} \bar{E}(t) d\theta = \alpha_R k [\rho + I(\theta, t) - g_N], \quad (29)$$

where $\bar{E}(t) = [E(t)/N(t)]$ is per capita world level of world expenditure.

Substitution of equations (3) and (11) into the first integral of equation (24) yields the following demand for manufacturing labor in Home:

$$\frac{E(t)}{\lambda} \int_0^{\tilde{\theta}} \frac{\alpha_Q^h(\theta)}{\alpha_Q^f(\theta)} d\theta. \quad (30)$$

⁵² To see this, one can rewrite equation (26) at steady-state as follows

$\frac{(\lambda - 1)E(t)}{\omega \lambda (\rho + I(\theta, t) - g_N)} = \alpha_R k N(t)$. The left hand-side of this last expression is smaller than the left hand-side of equation (27), while the right hand-side of this last expression is the same as the one in the right hand-side of equation (27). Therefore, the level of global R&D investment, I , has to be lower in equation (26) than it is in equation (27). As Home engages in R&D in more industries, its relative wage decreases (due to assumption A.1). The lower level of Home's relative wage will increase the level of global R&D investment, I .

Combining equations (15) and (17), one can obtain the demand for R&D labor in Home

$$kN(t)\alpha_n \int_0^{\tilde{\theta}} I^h(\theta, t) d\theta. \quad (31)$$

Given that there is a large number of independent industries, the law of large numbers implies that the integral in equation (31) can be written as follows:

$$\int_0^{\tilde{\theta}} I(\theta, t) d\theta = \tilde{\theta} I(t), \quad (32)$$

where $I(t) = \int_0^1 I(\theta, t) d\theta$ is the average "effective-R&D" of the world economy.

Substituting equations (30) and (31) (after taking into account equation (32)) into Home's full employment of labor condition (equation 24) yields the resource condition

$$\bar{N}^h(t) = \frac{\bar{E}(t)}{\lambda} \int_0^{\tilde{\theta}} \frac{\alpha_o^h(\theta)}{\alpha_o^f(\theta)} d\theta + k\alpha_n \tilde{\theta} I(t), \quad (33)$$

where $\bar{N}^h(t) = N^h(t)/N(t)$ is Home's population relative to the world population.

Similar substitutions yield the resource condition for Foreign:

$$\bar{N}^f(t) = \frac{(1 - \tilde{\theta})\bar{E}(t)}{\lambda} + k\alpha_n (1 - \tilde{\theta}) I(t), \quad (34)$$

where $\bar{N}^f(t) = N^f(t)/N(t)$ is Foreign's population relative to the world population.

The above resource conditions described by equations (33) and (34) hold at each instant in time because, by assumption, factor markets clear instantaneously in both countries. Equations (28), (29), (33), and (34) represent a system of four equations in four unknowns $\tilde{\theta}$, ω , I , and $\bar{E}$. Manipulating these four equations yields a second schedule in (θ, ω) space, the mutual resource schedule⁵³

⁵³ In Appendix C, I derive the mutual resource schedule and show that it is upward-sloping in (θ, ω) space.

$$\omega = \text{MR}(\tilde{\theta}) = \frac{\frac{\bar{N}^f}{(1-\tilde{\theta})} + k\alpha_R(\rho - g_N)\tilde{\theta}}{\bar{N}^h + \tilde{\theta}k\alpha_R(\rho - g_N)} \quad (35)$$

The mutual resource schedule states that the relative wage ω , which clears labor markets in both countries, is an increasing function of the range of goods $\tilde{\theta}$ produced in Home. If the range of goods produced by Home increases, Home's relative demand for labor (both in manufacturing and R&D) increases. The excess demand for labor drives the level of the relative wage higher.

The mutual resource condition (MR) can be depicted in Figure 4-1. The vertical axis measures Home's relative wage, ω , and the horizontal axis reflects the measure of industries, θ . The intersection of the downward sloping $\text{RD}(\tilde{\theta})$ schedule and the upward sloping $\text{MR}(\tilde{\theta})$ schedule at point E determines the steady-state equilibrium relative wage, ω , and the marginal industry $\tilde{\theta}$ in which both countries undertake production in goods and R&D services.⁵⁴ Therefore, I arrive at:

Proposition 1. *For sufficiently large $\bar{N}^f/\bar{N}^h$, there exists a unique steady-state equilibrium such that*

- (a) *Home's relative wage, ω , is greater than one,*
- (b) *Home has a sustained comparative advantage in the range of industries $\theta \in [0, \tilde{\theta})$. In each industry $\theta \in [0, \tilde{\theta})$, only Home conducts R&D, produces, and exports the state of the-art product,*

⁵⁴ For sufficiently large $\bar{N}^f/\bar{N}^h$, the $\text{MR}(\tilde{\theta})$ schedule intersects the $\text{RD}(\tilde{\theta})$ schedule at a point above the 45° line, such as $\omega > 1$.

- (c) *Foreign has a sustained comparative advantage in the range of industries $\theta \in (\tilde{\theta}, 1]$. In each industry $\theta \in (\tilde{\theta}, 1]$, only Foreign conducts R&D, produces, and exports the state of the-art product.*

Proof. See Appendix C.

The results from this proposition can be found in other models. The static continuum Ricardian model developed by Dornbusch et al. (1977) and the dynamic learning-by-doing model introduced by Krugman (1987) produce similar features with the equilibrium depicted in Figure 4-1. The pattern of trade in goods is determined by comparative advantage across industries. In addition and in contrast to earlier work, the model predicts that the pattern of trade is determined by additional factors such as population growth and the R&D difficulty parameter. Moreover, the absence of heterogeneity in research technologies results in no trade in R&D services. Taylor (1993) has introduced heterogeneity in research technologies and result in an equilibrium with trade in R&D services. Finally, factor price equalization is not a property of the equilibrium depicted in Figure 4-1.

4.4 Comparative Steady-State Analysis Under the PEG Specification

In this section I examine the comparative static properties of the steady-state equilibrium presented in Figure 4-1. By totally differentiating the equilibrium conditions (23) and (35), one can obtain propositions 2 and 3⁵⁵:

Proposition 2. *Assume that $\bar{N}^f / \bar{N}^h > (1 - \tilde{\theta}) / \tilde{\theta}$. Then under the PEG specification, Home's relative wage, ω ,*

⁵⁵ The results presented in this section are derived algebraically in Appendix C.

- (a) *depends positively on Foreign's relative size, $\bar{N}^f / \bar{N}^h$, Foreign's unit labor requirement in manufacturing, a_Q^f , the population growth rate, g_N ,*
- (b) *depends negatively on the R&D difficulty parameter, k , the unit R&D labor requirement, a_R , the consumer's subjective discount rate, ρ , and Home's unit labor requirement, a_Q^h ,*
- (c) *does not depend on the size of innovations, λ .*

Proof. See Appendix C.

The comparative steady-state properties of the PEG specification analysis are novel and differ from those in Taylor (1993). For example, in Taylor (1993), Home's relative wage depends on Foreign's relative size, Foreign's unit labor requirement in manufacturing, and Home's unit labor requirement. Proposition 2 identifies population growth and R&D difficulty parameter as important factors that affect the equilibrium relative wage.

The comparative steady-state results from the PEG specification can be illustrated graphically. Figure 4-3 shows that an increase in Foreign's relative size, $\bar{N}^f / \bar{N}^h$, shifts the MR schedule upward. At the initial relative wage equilibrium, ω , the increase in $\bar{N}^f / \bar{N}^h$ would create an excess supply of labor abroad and an excess demand for labor at home. The increase in relative wage restores the equilibrium at point E' . An increase in the population growth, g_N , operates through a decrease in the effective discount rate, $\rho - g_N$. It does not affect the RD schedule in Figure 4-3, but shifts the mutual resource schedule MR to the left, thus resulting in a higher relative wage, ω' . The increase in the population growth rate, g_N , increases the value of the expected discounted profits of a

successful innovator in both countries. At the same time, both countries become more innovative (the global R&D investment, $I(\theta, t)$, has to increase to maintain the equality in equation (27)). For a given increase in the global R&D investment, the relative wage has to increase in order to restore the equilibrium condition in equation (26).

An increase in the R&D difficulty parameter k , in the unit R&D labor requirement, α_R , or in the consumer's subjective discount rate, ρ , each shifts the MR schedule downward and decreases the relative wage (see Figure 4-4). Figure 4-5 shows that an increase in Foreign's unit labor requirement in manufacturing, α_Q^f , shifts the RD schedule upward and increases the relative wage.

Figure 4-6 shows that an increase in Home's unit labor requirement, α_Q^h , shifts the RD schedule downward and decreases the relative wage. Home's relative wage is left unaffected by equiproportionate changes in Home and Foreign production costs (the RD schedule is not shifted). Finally, changes in the size of innovations do not affect Home's relative wages. An increase in the size of innovations increases the profitability of R&D activity. As a result, more firms enter into R&D races until the expected profits are driven back to zero. The adjustment to equilibrium requires an increase in the level of global R&D investment, while it leaves Home's relative wage unchanged.

Proposition 3. Assume that $\bar{N}^f / \bar{N}^h > (1 - \tilde{\theta}) / \tilde{\theta}$. Then under the PEG specification, the range of goods Home produces, conducts R&D, and exports, $\tilde{\theta}$,

(a) depends positively on the R&D difficulty parameter, k , the unit R&D labor requirement, α_R , the consumer's subjective discount rate, ρ , Foreign's unit labor requirement in manufacturing, α_Q^f ,

- (b) *depends negatively on Foreign's relative size, $\bar{N}^f / \bar{N}^h$, the population growth rate, g_N , and Home's unit labor requirement in manufacturing, α_Q^h ,*
- (c) *does not depend on the size of innovations, λ .*

Proof. See Appendix C.

These comparative steady-state properties can also be illustrated graphically by utilizing Figures 4-3 through 4-6. In Figure 4-3 an increase in Foreign's relative size, $\bar{N}^f / \bar{N}^h$, shifts the MR schedule upward and reduces Home's comparative and absolute (if any) advantage in both goods and R&D production. In contrast to previous work, the pattern of goods and R&D production as well as the pattern of trade are also determined by population growth and the R&D difficulty parameter.

An increase in the population growth rate, g_N , shifts the MR schedule upward in Figure 4-3, and increases the relative wage. As a result, it reduces Home's comparative and absolute (if any) advantage in both goods and R&D production. An increase in the R&D difficulty parameter, k , in the unit R&D labor requirement, α_R , or in the consumer's subjective discount rate, ρ , each shifts the MR schedule downward and increase Home's comparative and absolute (if any) advantage in both goods and R&D production (see Figure 4-4).

Figure 4-5 shows that an increase in Foreign's unit labor requirement in manufacturing, α_Q^f , shifts the RD schedule upward and increases Home's comparative and absolute (if any) advantage in both goods and R&D production. Figure 4-6 shows that an increase in Home's unit labor requirement in manufacturing, α_Q^h , shifts the RD schedule downward and decreases Home's comparative and absolute (if any) advantage

in both goods and R&D production. Furthermore, Home's comparative advantage pattern is left unaffected by equiproportionate changes in Home and Foreign production costs (the RD schedule is not shifted).

Finally changes in the size of innovations do not affect Home's comparative advantage. An increase in the size of innovations increases the profitability of R&D activity. As a result, more firms enter into R&D races until the expected profits are driven back to zero. The adjustment to equilibrium requires an increase in the level of global R&D investment, while it leaves Home's comparative advantage unchanged.

The following proposition identifies several determinants of the global innovation rate, I , including its dependency on the population growth rate and the R&D difficulty parameter.

Proposition 4. *Under the PEG specification, the global R&D investment, I , depends*

- (a) *positively on Home's relative size measured by its share of world's population, $\bar{N}^h$, Foreign's relative size measured by its share of world's population, $\bar{N}^f$, the size of innovations, λ , the population growth rate, g_N ,*
- (b) *negatively on the R&D difficulty parameter, k , the consumer's subjective discount rate, ρ , and the unit R&D labor requirement, α_R .*

Proof. See Appendix C.

These results are standard in the endogenous growth literature without scale effects. An increase in either country's population size makes R&D more profitable and results in a higher level of R&D investment. An increase in the size of innovations, λ ,

increases the profit flows of the global incumbent firms (see equations (12) and (13)). As a result, the global innovation rate increases.

An increase in the rate of population growth, g_N , operates through a decrease in the effective discount rate, $\rho - g_N$. It raises the expected global discounted profits (see equations (26) and (27)) and thus it results in a higher global R&D investment. An increase in the R&D difficulty, k , or in the R&D unit labor requirement, α_R , makes R&D more costly and thus it reduces the level of global R&D investment. Finally, the increase in the consumer's subjective discount rate, ρ , lowers the global R&D investment.

4.5 Steady-State Equilibrium Under the TEG Specification

In this section I derive the steady-state equilibrium under the TEG specification proposed by Segerstrom (1998), which is described according to equation (18). It is easy to show that under the TEG specification, equation (23) still holds.⁵⁶

Combining equations (20) and (22), I obtain the following zero profit conditions for Home and Foreign respectively:

$$\frac{\left(1 - \frac{\omega \alpha_\phi^b(\theta)}{\alpha_\phi^f(\theta) \lambda}\right) \bar{E}(t)}{\rho + I(\theta, t) - g_N} = \omega \alpha_R k \bar{X}(t), \quad \forall \theta \in [0, \tilde{\theta}), \quad (38)$$

$$\frac{\left(\frac{\lambda - 1}{\lambda}\right) \bar{E}(t)}{\rho + I(\theta, t) - g_N} = \alpha_R k \bar{X}(t), \quad \forall \theta \in [\tilde{\theta}, 1], \quad (39)$$

where $\bar{X}(t) = X(t)/N(t)$ is the per-capita world level of R&D difficulty. In a steady-state equilibrium all per capita variables are constant. Therefore, the level of R&D difficulty

⁵⁶ In Appendix C, I provide the proof for the validation of equation (23) under the TEG specification.

grows at the same rate of population growth, $\dot{X}(t)/X(t) = \dot{N}(t)/N(t) = g_N$. This last result, combined with equation (16) yields

$$I = \frac{g_N}{\mu}. \quad (40)$$

Integrating equation (38) over $[0, \tilde{\theta}]$ and equation (39) over $(\tilde{\theta}, 1]$ (after taking into account equations (32) and (40)), I obtain the following zero profit conditions for Home and Foreign, respectively

$$\left(\tilde{\theta}\lambda - \omega \int_0^{\tilde{\theta}} \frac{\alpha_Q^b(\theta)}{\alpha_Q^f(\theta)} d\theta \right) \bar{E}(t) = \tilde{\theta}\omega\alpha_R \bar{X}(t)\lambda \left(\rho + \frac{g_N}{\mu} - g_N \right), \quad (41)$$

$$\frac{(\lambda - 1)}{\lambda} \bar{E}(t) = \alpha_R \bar{X}(t) \left(\rho + \frac{g_N}{\mu} - g_N \right). \quad (42)$$

The demand for manufacturing labor is still given by equation (30). Combining equations (15) and (17), the demand for R&D labor is given by the following expression

$$\frac{\alpha_R X(t) \tilde{\theta} g_N}{\mu}. \quad (43)$$

Substituting equations (30) and (43) (after taking into account equation (32)) into the home's full employment of labor condition yields the resource condition

$$\bar{N}^h(t) = \frac{\bar{E}(t)}{\lambda} \int_0^{\tilde{\theta}} \frac{\alpha_Q^b(\theta)}{\alpha_Q^f(\theta)} d\theta + \frac{\alpha_R g_N \bar{X}(t) \tilde{\theta}}{\mu}. \quad (44)$$

By similar substitutions, I can obtain the resource condition for Foreign:

$$\bar{N}^f(t) = \frac{(1 - \tilde{\theta})\bar{E}(t)}{\lambda} + \frac{\alpha_R g_N \bar{X}(1 - \tilde{\theta})}{\mu}. \quad (45)$$

Equations (41), (42), (44), and (45) represent a system of four equations in four

unknowns $\tilde{\theta}$, ω , $\bar{X}$, and $\bar{E}$. Manipulating these four equations yields a second schedule in (θ, ω) space, the mutual resource schedule⁵⁷

$$\omega = MR(\tilde{\theta}) = \frac{\bar{N}^f \lambda g_N \tilde{\theta}}{\bar{N}^h (1 - \tilde{\theta}) \left[\mu \left(\rho + \frac{g_N}{\mu} - g_N \right) + (\lambda - 1) g_N \right] - \bar{N}^f \left[\mu \left(\rho + \frac{g_N}{\mu} - g_N \right) - g_N \right] \int_0^{\tilde{\theta}} \frac{a_Q^h(\theta)}{a_Q^f(\theta)} d\theta}.$$

This last equation is referred to as equation (46).

The mutual resource schedule states that the relative wage ω , which clears labor markets in both countries, is an increasing function of the range of goods $\tilde{\theta}$ produced in Home. If the range of goods produced by Home increases, Home's relative demand for labor (both in manufacturing and R&D) increases. The excess demand for labor drives the level of the relative wage higher.

The mutual resource condition can be depicted in Figure 4-7. The intersection of the downward sloping $RD(\tilde{\theta})$ schedule and the upward sloping $MR(\tilde{\theta})$ schedule at point E determines the steady-state equilibrium relative wage, ω , and the marginal industry $\tilde{\theta}$ in which both countries undertake production in goods and R&D services. Therefore, I arrive at:

Proposition 5. *For sufficiently large $\bar{N}^f / \bar{N}^h$, there exists a unique steady-state equilibrium such that*

- (a) *Home's relative wage, ω , is greater than one,*

⁵⁷ In Appendix C, it is shown how the mutual resource condition is derived and that it is upward-sloping in (θ, ω) space.

- (b) *Home has a sustained comparative advantage in the range of industries $\theta \in [0, \tilde{\theta})$. In each industry $\theta \in [0, \tilde{\theta})$, only Home conducts R&D, produces, and exports the state of the-art product,*
- (c) *Foreign has a sustained comparative advantage in the range of industries $\theta \in (\tilde{\theta}, 1]$. In each industry $\theta \in (\tilde{\theta}, 1]$, only Foreign conducts R&D, produces, and exports the state of the-art product.*

Proof. See Appendix C.

As in the PEG specification analysis, the pattern of trade in goods is determined by comparative advantage across industries. Factor price equalization is also not a property of the equilibrium depicted in Figure 4-7. Finally, in contrast to the work of Taylor (1993), trade in R&D services does not occur.

4.6 Comparative Steady-State Analysis Under the TEG Specification

In this section I examine the comparative static properties of the steady-state equilibrium presented in Figure 4-7. By totally differentiating the equilibrium conditions (23) and (46) one can obtain propositions 6 and 7⁵⁸:

Proposition 6. *Assume that $\bar{N}^f / \bar{N}^h > (1 - \tilde{\theta}) \int_0^{\tilde{\theta}} \frac{\alpha_Q^h(\theta)}{\alpha_Q^f(\theta)} d\theta$. Then under the*

TEG specification, Home's relative wage, ω , depends

- (a) *positively on Foreign's relative size, $\bar{N}^f / \bar{N}^h$, Foreign's unit labor requirement in manufacturing, α_Q^f , the consumer's subjective discount rate, ρ , the R&D difficulty growth parameter μ ,*

⁵⁸ The results presented in this section are derived algebraically in Appendix C.

- (b) *negatively on the size of innovations, λ , the population growth rate, g_N , and the Home's unit labor requirement in manufacturing, a_0^h .*

Proof. See Appendix C.

These comparative steady-state properties can be illustrated graphically.

Figure 4-8 shows that an increase in Foreign's relative size, $\bar{N}^f/\bar{N}^h$, shifts the MR schedule upward. At the initial relative wage equilibrium, ω , the increase in $\bar{N}^f/\bar{N}^h$ creates an excess supply of labor abroad and an excess demand for labor at home. The increase in relative wages works as a mechanism to restore the equilibrium. These results are identical with those derived under the PEG specification.

Under the TEG model, an increase in the population growth rate, g_N , shifts the MR schedule downward in Figure 4-9 and decreases the relative wage from ω to ω' . The increase in the population growth rate, g_N , has two effects on the value of the expected discounted profits of a successful innovator in both countries (see equations (41) and (42)). First, the increase in the population growth rate, g_N , has a positive direct effect on the discounted expected global profits. Second, it has a negative indirect effect through the global innovation rate, I , (see equation (40)). Thus, if the later negative effect dominates the positive direct effect, then the relative wage must decrease to restore the equilibrium.

An increase in the consumer's subjective discount rate, ρ , or an increase in the R&D difficulty growth parameter μ , shifts the MR schedule upward and increases the relative wage as seen in Figure 4-8. An increase in Foreign's unit labor requirement in

manufacturing, α_Q^f , shifts the RD schedule upward and increases the relative wage (see Figure 4-10).

Figure 4-11 shows that an increase in Home's unit labor requirement in manufacturing, α_Q^h , shifts the RD downward and decreases the relative wage. Home's relative wage is left unaffected by equiproportionate changes in Home and Foreign production costs (the RD schedule is not shifted).

Finally, the TEG specification implies that the global level of R&D investment depends positively on the rate of population growth and negatively on the R&D difficulty parameter, since the R&D difficulty grows at the constant rate of population growth rate at the steady-state equilibrium. Therefore, an increase in the size of innovations raises the rewards of innovating and thus it induces more firms to engage in R&D. However, since the long-run level of R&D activity is fixed by the unchanged parameters of the model, to maintain the equilibrium, the relative wage has to decrease.

Proposition 7. Assume that $\bar{N}^f / \bar{N}^h > (1 - \tilde{\theta}) \left/ \int_0^{\tilde{\theta}} \frac{\alpha_Q^h(\theta)}{\alpha_Q^f(\theta)} d\theta \right.$. Then under the

TEG specification the range of goods Home produces, conducts R&D, and exports, $\tilde{\theta}$, depends

- (a) positively on the size of innovations, λ , the population growth rate, g_H , Foreign's unit labor requirement in manufacturing, α_Q^f ,
- (b) negatively on Foreign's relative size, $\bar{N}^f / \bar{N}^h$, the consumer's subjective discount rate, ρ , the R&D difficulty growth parameter μ , and Home's unit labor requirement in manufacturing, α_Q^h .

Proof. See Appendix C.

These comparative steady-state properties can also be illustrated graphically by utilizing Figures 4-8 through 4-11. In Figure 4-8 an increase in Foreign's relative size, $\bar{N}^f/\bar{N}^h$, shifts the MR schedule upward and reduces Home's comparative advantage in both goods and R&D production.

On contrast, an increase in the population growth rate, g_N , shifts the MR schedule downward in Figure 4-9, and decreases the relative wage. As a result, it increases Home's comparative and absolute (if any) advantage in both goods and R&D production.

An increase in the R&D difficulty parameter μ , or an increase in the consumer's subjective discount rate, ρ , each shifts the MR schedule upward and decreases Home's comparative and absolute (if any) advantage in both goods and R&D production (see Figure 4-8). Figure 4-10 shows that an increase in Foreign's unit labor requirement in manufacturing, α_Q^f , shifts the RD schedule upward and increases the Home's comparative and absolute (if any) advantage in both goods and R&D production.

Figure 4-11 shows that an increase in Home's unit labor requirement in manufacturing, α_Q^h , shifts the RD schedule downward and decreases Home's comparative and absolute (if any) advantage in both goods and R&D production. Finally, Home's comparative advantage pattern is left unaffected by equiproportionate changes in Home and Foreign production costs (the RD schedule is not shifted).

Proposition 8. *Under the TEG specification, the global R&D investment, I , depends positively on the R&D difficulty growth parameter, μ , and negatively on the population growth rate, g_N .*

Proof. It follows from equation (40).

The level of R&D investment, I , is completely determined by the exogenous rate of population growth $g_N > 0$ and the R&D difficulty growth parameter $\mu > 0$. The balanced-growth innovation rate is higher when the population of consumers grows more rapidly or when R&D difficulty increases more slowly over time. These results are standard in the endogenous growth literature without scale effects.⁵⁹

4.7 Conclusions

The previous literature on “quality ladders” framework that analyzed Ricardian models of trade exhibits the scale effects property. In this chapter, I have developed a model of trade based on “quality-ladders” growth without scale effects to analyze how the pattern of trade and the relative wage are determined in steady-state equilibrium. The absence of scale effects generates novel and interesting results.

The removal of scale effects property under either specification regarding the R&D difficulty leaves some predictions in Taylor’s (1993) model robust. A uniform increase in Home’s unit production costs reduces its relative wage and reduces its range of goods that Home produces. A uniform increase in Foreign’s unit production costs increases Home’s relative wage and increases Home’s comparative advantage. In addition, Home’s relative wage and its comparative advantage are left unaffected by equiproportionate changes in Home and Foreign production costs. Finally, factor price equalization is not a generic property of the steady-state equilibrium.

However, several comparative-steady-state results in Taylor’s (1993) model change with the removal of the scale effects property. First, in his model, the direction of

⁵⁹ See Segerstrom (1998).

the effect of the size of innovations on the pattern of goods production, R&D production, the pattern of trade, and the relative wage depends on the assumption that the size of innovations is heterogeneous. Removing part of this heterogeneity in his model, by eliminating Home's relative advantage in goods versus R&D, results in zero trade in R&D and no effect of the size of innovations on the pattern of trade and Home's relative wage. On contrast, in the present model, the direction of the effect of the size of innovations on the pattern of trade and Home's relative wage depends on the way in which the scale effects property is removed. Under the PEG specification, changes in the size of innovations do not affect Home's comparative advantage and its relative wage, while under the TEG specification, an increase in the size of innovations reduces Home's relative wage and raises its comparative advantage.

Under the PEG specification, the increase in the size of innovations increases the profitability of R&D activity. As a result, more firms enter into R&D races until the expected profits are driven back to zero. The adjustment to equilibrium requires an increase in the level of global R&D investment, while it leaves Home's relative wage and its comparative advantage unchanged. On contrast, the TEG specification implies that the global level of R&D investment depends positively on the rate of population growth and negatively on the R&D difficulty parameter, since the R&D difficulty grows at the constant rate of population growth rate at the steady-state equilibrium. Therefore, an increase in the size of innovations raises the rewards of innovating and thus it induces more firms to engage in R&D. However, since the long-run level of R&D activity is fixed by the unchanged parameters of the model, to maintain the equilibrium, the relative

wage has to decrease. The decrease in Home's relative wage raises its comparative advantage.

Second, the direction of the effect of a change in R&D costs in Taylor's (1993) model depends on the heterogeneous research assumption. Eliminating the heterogeneous research technologies assumption, results in factor price equalization in any equilibrium with both countries conducting R&D. In this case, an equiproportionate changes in R&D costs in his model, has no effect on Home's relative wage and its comparative advantage. Furthermore, if one assumes the absence of heterogeneous size of innovations and unity budget share for products in Taylor's model, then an equiproportionate changes in R&D costs makes Home conduct less R&D.

On contrast, in the present model, the direction of the effect of an equiproportionate change of the unit R&D labor requirement depends on the way in which the scale effects property is removed. Under the PEG specification, an increase in R&D costs reduces Home's relative wage and raises its comparative advantage, while under the TEG specification, changes in R&D costs do not affect Home's relative wage and its comparative advantage.

Under the PEG specification, the increase in R&D costs decreases the profitability of R&D activity. As a result, fewer firms enter into R&D races until the expected profits are driven back to zero. The adjustment to equilibrium requires a decrease in the level of Home's relative wage. The decrease in Home's relative wage raises its comparative advantage. On contrast, under the TEG specification, since the long-run level of R&D activity is fixed by the unchanged parameters of the model, an increase in R&D costs

increases the population-adjusted difficulty of R&D and thus leaving Home's relative wage and its comparative advantage unchanged.

The analysis in the present model generates new additional findings. Under both specifications, the model generates a unique steady-state equilibrium in which there is complete specialization in both goods and R&D production within each industry. Trade between the two countries occurs only in goods and not in R&D services. In contrast to the work of Grossman and Helpman (1991*c*), factor price equalization does not hold in the steady-state equilibrium under either specification.

Under the PEG specification, Home's relative wage depends positively on Foreign's relative size, Foreign's unit labor requirement in manufacturing, and the population growth rate. It depends negatively on the R&D difficulty parameter, the unit R&D labor requirement, the consumer's subjective discount rate, and Home's unit labor requirement in manufacturing. In contrast to previous models (Grossman and Helpman (1991*c*), and Taylor (1993)), these results highlight the effects of population growth and the R&D difficulty on relative wages.

Under the PEG specification, the range of goods produced in Home and exported depends positively on the R&D difficulty parameter, the unit R&D labor requirement, the consumer's subjective discount rate, and Foreign's unit labor requirement in manufacturing. It depends negatively on Foreign's relative size, the population growth rate, and Home's unit labor requirement in manufacturing. These results also highlight the importance of population growth and the R&D difficulty parameter on the pattern of goods and R&D production, and the pattern of trade between the two countries.

Some results that hold under the PEG specification analysis are reversed when the TEG specification is introduced in the model. Home's relative wage depends positively on the consumer's subjective discount rate and the R&D difficulty growth parameter. It depends negatively on the population growth rate. The range of goods Home produces and exports depends positively on the population growth rate. It depends negatively on the consumer's subjective discount rate and the R&D difficulty growth parameter.

The incorporation of population growth and the removal of scale effects highlight the importance of population growth, consumer's subjective discount rate, and the R&D difficulty parameter on goods production, R&D production, and trade pattern between the two countries. The model explores its comparative steady state properties of equilibria under two alternative specifications regarding the R&D difficulty, the PEG and the TEG specification. I find that the direction of the effects of population growth rate and consumer's subjective discount rate on Home's relative wage, the pattern of goods and R&D production, and the pattern of trade between the two countries depends on whether the PEG or the TEG R&D specification is assumed.

Given the relatively simplicity of the model, this dynamic formulation provides a useful framework to examine other issues. For example, the introduction of trade instruments and their effect on the pattern of trade between countries can be examined under the two alternative models. Alternatively, a North-South model of trade might yield interesting implications.

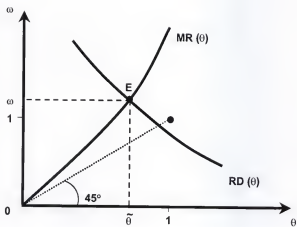


Figure 4-1. Steady-State Equilibrium Under the PEG Specification

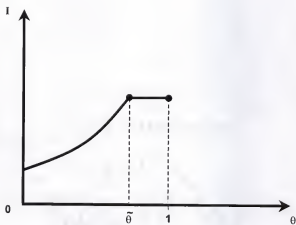


Figure 4-2. Ranking of the Global R&D Investment, I , Between Home and Foreign, Under the PEG Specification

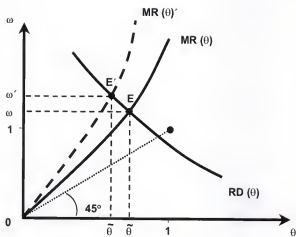


Figure 4-3. Effects of an Increase in the Foreign County's Relative Size, $\bar{N}'/\bar{N}^h$, or in the Population Growth Rate, g_N (Under the PEG Specification)

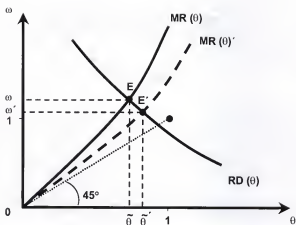


Figure 4-4. Effects of an Increase in the R&D Difficulty Parameter, k , in the Unit R&D Labor Requirement, α_R , or in the Consumer's Subjective Discount Rate, ρ (Under the PEG Specification)

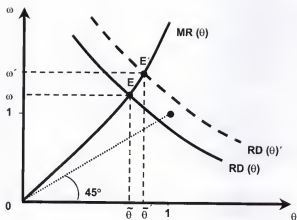


Figure 4-5. Effects of an Increase in the Foreign's Unit Labor Requirement in Manufacturing, α_Q^f (Under the PEG Specification)

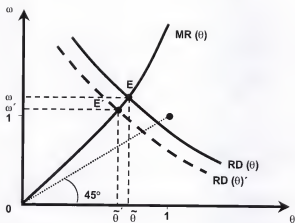


Figure 4-6. Effects of an Increase in Home's Unit Labor Requirement in Manufacturing, a_Q^h (Under the PEG Specification)

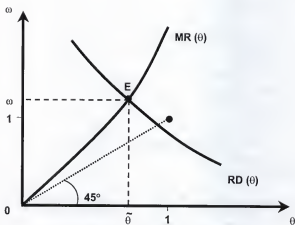


Figure 4-7. Steady-State Equilibrium Under the TEG Specification

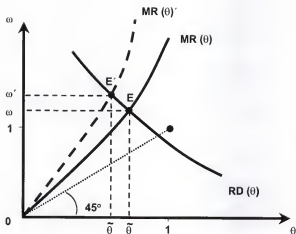


Figure 4-8. Effects of an Increase in the Foreign Country's Relative Size, $\bar{N}^f/\bar{N}^h$, in the Consumer's Subjective Discount Rate, ρ , or in the R&D Difficulty Growth Parameter, μ (Under the TEG Specification)

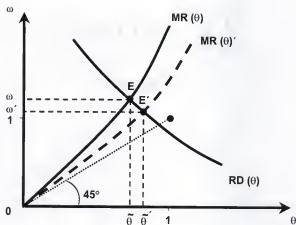


Figure 4-9. Effects of an Increase in the Size of Innovations, λ , or in the Population Growth Rate, g_N (Under the TEG Specification)

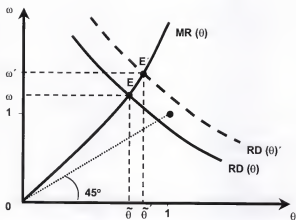


Figure 4-10. Effects of an Increase in Foreign's Unit Labor Requirement in Manufacturing, α_Q^f (Under the TEG Specification)

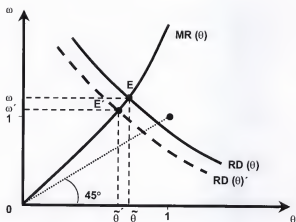


Figure 4-11. Effects of an Increase in Home's Unit Labor Requirement in Manufacturing, a_Q^h (Under the TEG Specification)

CHAPTER 5 CONCLUSION

This dissertation has analyzed several issues involving the economics of endogenous technological progress: the effects of firm size on the choice of R&D effort between process and product innovation, the transitional and long-run dynamic effects of general-purpose technologies on Schumpeterian growth in a closed economy setting, and the interactions between endogenous technological change, the pattern of trade, and income distribution within each country.

The differentiated-goods duopoly model developed in Chapter 2 confirms the positive relationship between firm size and the fraction of process versus product R&D that firms undertake. My model shows that an increase in a firm's efforts devoted to product innovation, given that it is in the product R&D regime, increases its incentives to switch from product to process innovation.

My findings support the basic idea that larger firms have an advantage in R&D because of the larger output over which they can apply the results-and thus spread the costs-of their R&D (Cohen and Klepper (1996)). Note, however, that once a firm switches from product to process, it continues to develop only process (since the size of process innovation in my model is greater than one). If the size of process innovation were small, by performing one more product R&D in the process R&D regime would increase the incentives to switch from process to product innovation. Indeed, if the size of process innovation is less than one, process innovation is not profitable.

Two limitations of the model need to be brought to the forefront. First, my conclusions depend on and are limited by the functional forms and the assumptions of my model. Another production function, for example, can make the model conform better to the real world. That is, once a firm is in the process R&D regime, after a critical number of process innovations, it can be possible to switch from process to product innovations. Second, the assumption that firms will not attend to the production process until product innovation has slowed sufficiently is also restrictive. Yet the history of the automobile industry and others, such as tires and antibiotics, indicates that great improvements were made in the production process well before the emergence of any kind of dominant design. Indeed, many of these improvements were based on human and physical improvements that were not rendered obsolete by subsequent major product innovations. One possible extension of the model is to relax this assumption and explore the implications of this change on firm's optimal decision on choosing the fraction of process and product innovation. This relaxation would change some of the results, but it would also complicate the analysis of the model.

Previous models that have analyzed GPTs exhibit the scale effects property. Chapter 3 analyzed the effects of a GPT on short-run and long-run Schumpeterian growth without scale effects. The absence of growth scale effects and the modeling of the diffusion process through *S*-curve dynamics generated several novel and interesting results.

First, the long-run growth rate of the economy depends positively on the magnitude of quality innovations. Any policy that affects this magnitude has long-run growth effects. However, the absence of the arrival of a new GPT in the economy does

not reduce the long-run growth rate to zero as in the previous GPTs-based growth models. All the previous R&D-based models that analyze the effects of GPTs exhibit scale effects.

The assumption that the diffusion of the new GPT follows an *S*-curve generates two steady-state equilibria: one is the initial steady-state before the adoption of the new GPT begins and the other is the final steady-state after the diffusion process of the new GPT has been completed. At the final steady-state relative to the initial steady-state: the long-run growth rate is higher, the aggregate investment is higher, the per capita consumption expenditure is lower, and the market interest rate is equal to the subjective discount rate.

The growth rate of the stock market depends negatively on the rate of GPT diffusion process, and the magnitude of the GPT-ridden R&D productivity gains, and positively on the rate of population growth. It also follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT. This is consistent with the empirical evidence provided by Jovanovic and Rousseau (2000) who empirically document that during times of rapid technological change the growth of the stock market slows down, since the new entrants of the stock market will grab the most value from previous entrants because the incumbents will find hard to keep up.

One could also develop a dynamic general equilibrium model to study the effects of a GPT diffusion on a global economy that exhibits endogenous Schumpeterian growth. As in this model, the adoption of a GPT by a particular industry can generate an increase in the productivity of R&D workers and the magnitude of all future innovations and its diffusion across industries can be governed by *S*-curve dynamics. The diffusion of the

GPT within an industry from one country to the other can occur with a time lag. Under this framework, it would be interesting to analyze the long-run and transitional dynamic effects of a new GPT on trade patterns, product cycles and (transitional) divergence in per-capita growth rates between the two countries. This is a fruitful direction for future research.

Chapter 4 constructed a two-country (Home and Foreign) general equilibrium model of Schumpeterian growth without scale effects. The scale effects property is removed by introducing two distinct specifications in the knowledge production function: The permanent effect on growth (PEG) specification which allows policy effects on long-run growth; and the temporary effects on growth (TEG) specification, which generates exogenous long-run economic growth.

The removal of scale effects property under either specification regarding the R&D difficulty leaves some predictions in Taylor's (1993) model robust. For example, a uniform increase in Home's unit production costs reduces its relative wage and reduces its range of goods that Home produces. On the other hand, Home's relative wage and its comparative advantage are left unaffected by equiproportionate changes in Home and Foreign production costs.

The incorporation of population growth and the removal of scale effects highlight the importance of population growth, consumer's subjective discount rate, and the R&D difficulty parameter on goods production, R&D production, and trade pattern between the two countries. The comparative static properties of the model depend on the way that the scale effects property is removed. Under the PEG specification, changes in the size of innovations do not affect relative wages and comparative advantage, while under the

TEG specification, an increase in the size of innovations reduces Home's relative wage and raises its comparative advantage. In addition, under the PEG specification, an increase in the rate of population growth raises Home's relative wage and decreases its range of goods exported to Foreign. On the other hand, under the TEG specification, an increase in the rate of population growth rate lowers Home's relative wage and increases its range of goods exported to Foreign.

APPENDIX A PROOFS OF PROPOSITIONS IN CHAPTER 2

A.1 Proofs of Proposition 1, 2, 3, and 4

A.1.1 Proposition 1

The profit function for firm 1 for producing n_1 goods is given by:

$$\pi_1 = \sum_{i=1}^{n_1} P_i D_i - \frac{1}{\mu^{q_1}} \sum_{i=1}^{n_1} D_i. \quad (\text{A.1})$$

By substituting equation (2) into equation (A.1), I obtain the following:

$$\pi_1 = \sum_{i=1}^{n_1} (\alpha D_i^* - \frac{1}{\mu^{q_1}} D_i). \quad (\text{A.2})$$

The necessary conditions for a maximum are the following:

$$\frac{\partial \pi_1(D_i^*)}{\partial D_i} = 0, \quad \forall i = 1, \dots, n_1.$$

These necessary conditions imply that

$$D_i^* = \left(\frac{1}{\alpha^2 \mu^{q_1}} \right)^{1/(\alpha-1)}, \quad \forall i = 1, \dots, n_1, \quad (\text{A.3})$$

where asterisks denote optimal values.

The second-order conditions for a maximum are satisfied and given by the following:

$$\frac{\partial^2 \pi_1(D_i^*)}{\partial D_i^2} = \alpha^2 (\alpha - 1) D_i^{*\alpha-2} < 0, \quad \forall i = 1, \dots, n_1, \quad (\text{A.4})$$

since $\alpha < 1$.

The value profits can be found by substituting the equations for differentiated goods (equation (A.3) into equation (A.2)):

$$\begin{aligned}
\pi_1^* &= \sum_{i=1}^{n_1} \left[\alpha \left(\frac{1}{\alpha^2 \mu^{q_1}} \right)^{\alpha/(n-1)} - \frac{1}{\mu^{q_1}} \left(\frac{1}{\alpha^2 \mu^{q_1}} \right)^{1/(n-1)} \right] \\
&= n_1 \left[\left(\frac{1}{\mu^{q_1}} \right)^{\alpha/(n-1)} \left(\frac{1}{\alpha^2} \right)^{1/(n-1)} \left(\alpha \frac{1}{\alpha^2} - 1 \right) \right] \\
&= n_1 \left[\left(\frac{1}{\mu^{q_1}} \right)^{\alpha/(n-1)} (1 - \alpha) \left(\frac{1}{\alpha} \right)^{(n+1)/(n-1)} \right] \\
&= n_1 \left(\frac{1}{\mu^{q_1}} \right)^{\alpha/(n-1)} \frac{(1 - \alpha)}{\alpha^{(n+1)/(n-1)}} > 0, \quad (\text{since } \alpha < 1). \quad (A.5)
\end{aligned}$$

In the same way, the profit function for firm 2 for producing n_2 goods, is given by the following:

$$\begin{aligned}
\pi_2 &= \sum_{i=n_1+1}^N P_i D_i - \frac{1}{\mu^{q_2}} \sum_{i=n_1+1}^N D_i \\
&= \sum_{i=n_1+1}^N \left(\alpha D_i^\alpha - \frac{1}{\mu^{q_2}} D_i \right).
\end{aligned}$$

The necessary conditions for a maximum are the following:

$$\frac{\partial \pi_2(D_i^*)}{\partial D_i} = 0, \quad \forall i = n_1 + 1, \dots, N.$$

These conditions imply that

$$D_i^* = \left(\frac{1}{\alpha^2 \mu^{q_2}} \right)^{1/(n-1)}, \quad \forall i = n_1 + 1, \dots, N.$$

The second-order conditions for a maximum are satisfied and given by the following:

$$\frac{\partial^2 \pi_2(D_i^*)}{\partial D_i^2} = \alpha^2 (\alpha - 1) D_i^{*\alpha(n-2)} < 0, \quad \forall i = n_1 + 1, \dots, N,$$

since $\alpha < 1$.

The maximum profits for firm 2 are the following:

$$\pi_2^* = n_2 \left(\frac{1-\alpha}{\alpha^{a+1/a-1}} \right) \left(\frac{1}{\mu^{q_2}} \right)^{a/(a-1)} > 0, \quad (\text{A.6})$$

Since equations (A.5) and (A.6) are symmetric, I can generalize them by using the following equation:

$$\pi_k^* = n_k \left(\frac{1-\alpha}{\alpha^{a+1/a-1}} \right) \left(\frac{1}{\mu^{q_k}} \right)^{a/(a-1)} > 0, \quad (\text{A.7})$$

for $k = 1, 2$ (where k denotes either firm).

Next, I can do some comparative statics. Since n_k is a discrete variable, I evaluate equation (A.7) at n_k and $n_k + 1$ for a given q_k , and then I take the difference between them as follows:

$$\Delta \pi_{k,n_k}^* = \pi_k^*(n_k + 1, q_k) - \pi_k^*(n_k, q_k) = \left(\frac{1-\alpha}{\alpha^{a+1/a-1}} \right) \left(\frac{1}{\mu^{q_k}} \right)^{a/(a-1)} > 0, \quad (\text{A.8})$$

where $\Delta \pi_{k,n_k}^*$ is the marginal profit of firm k from product innovation.

Since q_k is also a discrete variable, I follow the same procedure as in the above case:

$$\begin{aligned} \Delta \pi_{k,q_k}^* &= \pi_k^*(q_k + 1, n_k) - \pi_k^*(q_k, n_k) = \left[n_k \left(\frac{1-\alpha}{\alpha^{a+1/a-1}} \right) \mu^{(q_k+1)a/(1-a)} - n_k \left(\frac{1-\alpha}{\alpha^{a+1/a-1}} \right) \mu^{q_k a/(1-a)} \right] \\ &= n_k \left(\frac{1-\alpha}{\alpha^{a+1/a-1}} \right) \mu^{q_k a/(1-a)} (\mu^{a/(1-a)} - 1) > 0, \end{aligned} \quad (\text{A.9})$$

where $\Delta \pi_{k,q_k}^*$ is the marginal profit of firm k from process innovation and k denotes firm ($k = 1, 2$).

The signs of equations (A.8) and (A.9) prove Proposition 1.

A.1.2 Proposition 2

By taking the difference between (A.9) and (A.8) and setting it to zero, I obtain the following:

$$\Delta \pi_{k,qk}^* - \Delta \pi_{k,nk}^* = n_k \left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}} \right) \mu^{q_k \alpha / (1-\alpha)} (\mu^{n/(1-\alpha)} - 1) - \left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}} \right) \left(\frac{1}{\mu^{q_k}} \right)^{\alpha/(s-1)} = 0.$$

From the above equation, I can solve for n_k^* as follows:

$$n_k^* = (1/\mu^{q_k})^{\alpha/(s-1)} [1/\mu^{\alpha q_k/(1-\alpha)} (\mu^{n/(1-\alpha)} - 1)]. \quad (\text{A.10})$$

Equation (A.10) proves Proposition 2.

Firm k will perform process innovation rather than product innovation if and only if

$$\Delta \pi_{k,qk}^* - \Delta \pi_{k,nk}^* > 0.$$

This last expression implies that

$$n_k^* > (1/\mu^{q_k})^{\alpha/(s-1)} [1/\mu^{\alpha q_k/(1-\alpha)} (\mu^{n/(1-\alpha)} - 1)].$$

A.1.3 Proposition 3

The firm k is in the product R&D regime if and only if

$$\begin{aligned} \Delta \pi_{k,qk}^* - \Delta \pi_{k,nk}^* &= \\ &= \frac{1-\alpha}{\alpha^{(s+1)/(s-1)}} (\mu^{n/(1-\alpha)} - 1) \mu^{\alpha q_k/(1-\alpha)} [n_k (\mu^{n/(1-\alpha)} - 1) - 1] < 0, \end{aligned} \quad (\text{A.11})$$

$$\text{or } n_k^* < 1/(\mu^{\alpha/(1-\alpha)} - 1).$$

By differentiating equation (A.11) with respect to n_k , I obtain the following:

$$\Delta(\Delta \pi_{k,qk}^* - \Delta \pi_{k,nk}^*)_{n_k} = \mu^{\alpha q_k/(1-\alpha)} \left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}} \right) (\mu^{n/(1-\alpha)} - 1) > 0, \quad (\text{A.12})$$

since $\mu > 1$.

The sign of equation (A.12) proves Proposition 3.

A.1.4 Proposition 4

The net marginal profit of firm k from process innovation is given by:

$$n_k \left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}} \right) \mu^{q_k \alpha/(1-\alpha)} (\mu^{n/(1-\alpha)} - 1) - R_{ck}. \quad (\text{A.13})$$

The net marginal profit of firm k from product innovation is given by:

$$\left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}}\right)\left(\frac{1}{\mu^{a_k}}\right)^{\alpha/(1-\alpha)} - R_{pk}. \quad (\text{A.14})$$

By equalizing (A.13) and (A.14), I can calculate the number of goods (n_k^*), at which the firm k is indifferent between performing one more product innovation and performing one more process innovation:

$$n_k \left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}}\right) \mu^{a_k \alpha/(1-\alpha)} (\mu^{a_k/(1-\alpha)} - 1) - R_{ck} = \left(\frac{1-\alpha}{\alpha^{(s+1)/(s-1)}}\right) \left(\frac{1}{\mu^{a_k}}\right)^{\alpha/(1-\alpha)} - R_{pk}.$$

The last expression implies

$$n_k^* = 1/(\mu^{a_k/(1-\alpha)} - 1) - \left(\alpha^{(s+1)/(s-1)}(R_{pk} - R_{ck})/(1-\alpha)\mu^{a_k/(1-\alpha)}(\mu^{a_k/(1-\alpha)} - 1)\right). \quad (\text{A.15})$$

Equation (A.15) proves Proposition 4.

A.1.5 Proposition 5

By taking the derivative of n_k^* in equation (A.15) with respect to the R&D cost from product innovation, R_{pk} , we obtain the following:

$$\frac{\partial n_k^*}{\partial R_{pk}} = -\frac{\alpha^{(s+1)/(s-1)}}{(1-\alpha)\mu^{a_k/(1-\alpha)}(\mu^{a_k/(1-\alpha)} - 1)} < 0, \quad (\text{A.16})$$

since $\mu > 1$ and $\alpha < 1$, both the numerator and the denominator are positive.

The sign of the equation A.16 proves Proposition 5.

A.1.6 Proposition 6

By taking the derivative of n_k^* in equation (A.15) with respect to the R&D cost from process innovation, R_{ck} , we obtain the following:

$$\frac{\partial n_k^*}{\partial R_{ck}} = \frac{\alpha^{(s+1)/(s-1)}}{(1-\alpha)\mu^{a_k/(1-\alpha)}(\mu^{a_k/(1-\alpha)} - 1)} > 0, \quad (\text{A.17})$$

since $\mu > 1$ and $\alpha < 1$, both the numerator and the denominator are positive.

The sign of the equation (A.17) proves Proposition 6.

A.1.7 Proposition 7

First, I calculate the difference between the marginal profit from process innovations and the marginal profit from product innovations as follows:

$$\begin{aligned}\Delta\pi_{k,q_k}^* - \Delta\pi_{k,s_k}^* &= n_k \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) \mu^{a_k/(1-\alpha)} (\mu^{a/(1-\alpha)} - 1) - \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) \left(\frac{1}{\mu^{q_k}} \right)^{a/(a-1)} \\ &= n_k \mu^{a(q_k-1)/(1-\alpha)} \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) (\mu^{a/(1-\alpha)} - 1) - \left(\frac{1}{\mu^{q_k}} \right)^{a/(a-1)} \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) - \\ &\quad - n_k \mu^{a q_k/(1-\alpha)} \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) (\mu^{a/(1-\alpha)} - 1) + \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) \left(\frac{1}{\mu^{q_k}} \right)^{a/(a-1)} \\ &= \frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} (\mu^{a/(1-\alpha)} - 1) \mu^{a q_k/(1-\alpha)} [n_k (\mu^{a/(1-\alpha)} - 1) - 1],\end{aligned}\quad (\text{A.18})$$

since $\mu > 1$ and $\alpha < 1$, the sign of equation (A.18) depends on the sign of the expression in the brackets:

$$[n_k (\mu^{a/(1-\alpha)} - 1) - 1].$$

Firm k is in the process R&D regime if and only if

$$\Delta\pi_{k,q_k}^* - \Delta\pi_{k,s_k}^* = \frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} (\mu^{a/(1-\alpha)} - 1) \mu^{a q_k/(1-\alpha)} [n_k (\mu^{a/(1-\alpha)} - 1) - 1] > 0.$$

This last expression implies $[n_k (\mu^{a/(1-\alpha)} - 1) - 1] > 0$. (A.19)

By developing one more process innovation, the difference between the marginal profit from process and the marginal profit from product innovation increases by:

$$\Delta(\Delta\pi_{k,q_k}^* - \Delta\pi_{k,s_k}^*)_{q_k} = \left(\frac{1-\alpha}{\alpha^{(a+1)/(a-1)}} \right) (\mu^{a/(1-\alpha)} - 1) \mu^{a q_k/(1-\alpha)} [n_k (\mu^{a/(1-\alpha)} - 1) - 1], \quad (\text{A.20})$$

where $\Delta(\Delta\pi_{k,q_k}^* - \Delta\pi_{k,s_k}^*)_{q_k}$ is the change in the difference between the marginal profit from process and the marginal profit from product innovation due to change in the

number of process innovations by one. Given that firm k is in the process R&D regime, equation (A.19) holds, which implies that the sign of equation (A.20) is positive. That is, the firm has an incentive to continue to perform process R&D.

By developing one more product innovation, the difference between the marginal profit from process and the marginal profit from product innovation increases by:

$$\Delta(\Delta\pi_{k,n_k}^* - \Delta\pi_{k,n_k}^*)_{n_k} = \mu^{a_{n_k}/(1-a)} \left(\frac{1-a}{a^{(a+1)/(1-a)}} \right) (\mu^{a/(1-a)} - 1) > 0. \quad (\text{A.21})$$

The signs of equations (A.20) and (A.21) prove Proposition 7.

APPENDIX B

SUMMARY OF THE HISTORICAL EVOLUTION OF SEMICONDUCTOR TECHNOLOGY AND PROOFS OF PROPOSITIONS IN CHAPTER 3

B.1 The Evolution of Semiconductor Technology

In December 1947, John Bardeen and Walter Brattain, at Bell Laboratories, invented a new GPT, the transistor, a small electronic device containing a semiconductor that contains three electrical contacts. The transistor is made up from materials such as steel and copper that allows electrical current to flow and it is used in a circuit as an amplifier, detector, or switch. This invention began the semiconductor era. It was first adopted by the hearing aids sector in the early 1950s because of its amplification abilities and it was not long after when it replaced the vacuum tube, a system of electrodes arranged in an evacuated glass or metal envelope, in portable radios in 1954. Another early adopter of the transistor was the computer in 1955. This greatly changed the computer's development by also replacing the large, cumbersome vacuum tube, and making computers smaller, faster, more reliable, and more energy-efficient than before. After the introduction of the transistor portable radio in the late 1950s, Radio Corporation of America, tried to allow the transistor to be used in automobiles later in the 1970s. It included an antenna and a special bracket for the dashboard and was responsible for the mass infiltration of the recording industry into mainstream society. The automobile sector, and ironically the telecommunications sector (1990s) were considered as laggards because they were the last sectors to adopt this new GPT, the transistor.

Though transistors were clearly an improvement over the vacuum tube, they still generated a great deal of heat, which damaged the computer's sensitive internal parts. The computer, another GPT, became smaller with the development of the integrated circuit in 1958. The integrated circuit combined three electronic components, such as the transistor, onto a small silicon disc made from quartz, which eliminated the heat generation problem. After the integrated circuits, the only place to go was down in size. Large scale integration was developed in the 1970s, which could fit hundreds of components, such as the transistor, onto one silicon chip. By the 1980s, very-large scale integration squeezed hundreds of thousands of components onto a chip and ultra-large scale integration increased that number into the millions. The ability to fit so much onto an area about half the size of a U.S. dime helped diminish the size, increase the power, increase efficiency and reliability, and price of computers.⁶⁰

In 1981, IBM introduced the personal computer for use in the home, office, and schools. The Intel 4004 chip, developed in 1971, took the integrated circuit one step further by locating all components of a computer such as the central processing unit, memory, and input and output controls on a minuscule chip. Smaller than a thumbnail and packing 2300 transistors, the \$200 chip delivered as much computing power as the first electronic computer, ENIAC. Previously, the integrated circuit had to be manufactured to fit a special purpose, now one microprocessor could be manufactured and then programmed to meet any number of demands in items such as microwave ovens, television sets, and automobiles. In 1993, Intel introduced the Pentium processor,

⁶⁰ As Moore notes, "If the auto industry advanced as rapidly as the semiconductor industry, a Rolls Royce would get a half a million miles per gallon, and it would be cheaper to throw it away than to park it" (Intel Corporation, 2001).

an incremental innovation, which set new performance standards and used 3.1 million transistors to perform up to 90 millions of instructions per second, about 1,500 times the speed of the original 4004 (Jones International, 1999, Computers: History and Development, <http://www.digitalcentury.com>). An incremental innovation is small improvements and extensions to existing products, processes, or services. The target with incremental innovations is to increase efficiency, higher quality, reduced times, and lower costs.

As computers became more widespread in the workplace, ways to harness their potential developed. As smaller computers became more powerful, they could be linked together, or networked, to share memory space, software, information, and communicate with each other. With this global web of computer circuitry, the Internet, another GPT, became in use in the 1990s and linked computers worldwide into a single network of information. Using tools like electronic mail, mailing lists, video conferencing, and file transferring, the human network of any individual can be expanded to include others with similar interests all around the world. These communication tools allow us to share ideas, collaborate on projects, ask questions, and request information from experts, researchers, professionals, and enthusiasts in a wide variety of fields.

The Internet and the associated technology are changing the nature of computing. Fiber optics has helped push the telecommunications system into hyperdrive. But only when fiber connections reach all the way into the home will the technology's promise be fully realized. Fiber optics will soon take over copper cables and allow the link between communication, such as the Internet, to be always on, just like electricity in a socket, and information to flow at speeds that leave dialup modems in the dust. Experts say that

unprecedented growth in business and consumer demand for high-speed fiber-optic links and voice-over-Internet protocol will continue.

Many advances in the science of computer design and technology are coming together. As we rush towards the limits of traditional chip manufacturing technologies, entirely new approaches are beginning to be produced in laboratories. Three-dimensional chip arrays will bring another dimension to circuit layout. Deoxyribonucleic Acid (DNA) computing could be an immensely powerful form of parallel processing that could illustrate the possibilities of using DNA to solve a class of problems that is difficult or impossible to solve using traditional computing methods at a size limit that may never be reached by the semiconductor industry. Quantum computing is showing real results and could eventually lead to computers many orders of magnitude more powerful than today's supercomputers. Nanocomputers are also advancing in today's technologies. These basic devices are an important step towards developing tiny computers that will be faster, cheaper and more powerful than anything available with today's technology. These and other emerging computing technologies suggest that the exponential growth in computer power and technology will continue for decades to come.

B.2 Proofs of Propositions in Chapter 3

B.2.1 Proposition 1

Equations (27) and (29) define a unique steady-state equilibrium. The long-run Schumpeterian growth rate is endogenous and does not exhibit scale effects. This follows from equation (23).

Solving the expression for I in (27) (by substituting equation (27) into equation (28) in the main text), substituting it in (23) and differentiating the resulting expression with respect to the appropriate parameter.

At each steady-state, per capita consumption expenditure is constant. That is $\dot{c} = 0$. Then equation (8) implies that the interest rate, $\hat{r}$, is equal to the subjective discount rate, ρ .

The aggregate stock value is given by the following equation:

$$\bar{V} = [\omega \frac{V_1}{N(t)} + (1 - \omega) \frac{V_0}{N(t)}] N(t), \quad (\text{B.1})$$

where V_1 and V_0 are given by equation (20) after taking account equation (26). After substitution of these values into equation (B.1) and taking logs and derivatives with respect to time, I obtain the growth rate of the aggregate stock value as follows:

$$g_V = \frac{\dot{\bar{V}}}{\bar{V}} = g_N - \frac{(\mu - 1)\omega\delta(1 - \omega)}{[\omega + (1 - \omega)\mu]}. \quad (\text{B.2})$$

At the initial steady-state, where $\omega = 0$ and at the final steady-state, where $\omega = 1$, the growth rate of the aggregate stock value is equal to the rate of the population growth. That is

$$\hat{g}_V = g_N. \quad (\text{B.3})$$

This completes the proof of Proposition 1.

B.2.2 Proposition 2

Equations (1) and (31) define two loci: one where $\omega = 0$ and one where $\omega = 1$.

The aggregate investment is given by

$$I = I_0(1 - \omega) + I_1\omega. \quad (\text{B.4})$$

Evaluating the aggregate investment at the two steady-states, I obtain the following:

$$\hat{I} = \begin{cases} \hat{I}(0) = \hat{I}_0 & \text{for } \omega = 0 \\ \hat{I}(1) = \hat{I}_1 & \text{for } \omega = 1. \end{cases} \quad (\text{B.5})$$

Combining equations (18) and (20), I can solve for the innovations rates for both types of industries

$$I_1(\omega) = \frac{\mu}{k\alpha_R} \left(\frac{\lambda_1 - 1}{\lambda_1} \right) c(\omega) + g_N - r(\omega). \quad (\text{B.6})$$

$$I_0(\omega) = \frac{1}{k\alpha_R} \left(\frac{\lambda_0 - 1}{\lambda_0} \right) c(\omega) + g_N - r(\omega). \quad (\text{B.7})$$

Equations (B.6) and (B.7) imply that $I_1 > I_0$.

Thus, equations (B.5), (B. 6), and (B.7) imply that the aggregate investment is higher at the final steady state relative to the initial steady state.

Substituting equations (6) and (10) into equation (4) in the main text (after taking account equation (26)), I obtain the following:

$$\log u(t) = \log c(t) - \log \alpha_Q + \int_0^1 \log \lambda_i(\theta)^{x(\theta)} d\theta - \int_0^1 \log \lambda(\theta) d\theta. \quad (\text{B.8})$$

I calculate the first integral in equation (B.8) as follows:

$$\int_0^1 \log \lambda_i(\theta)^{x(\theta)} d\theta = \int_0^\infty \log \lambda_1^j d\theta + \int_\infty^1 \log \lambda_0^j d\theta. \quad (\text{B.9})$$

The first integral in the right-hand side of equation (B.9) is equal to the following:

$$\int_0^\infty \log \lambda_1^j d\theta = \int_0^\infty \sum_{m=0}^{\infty} [f(m, t) \log \lambda_1^m] d\theta. \quad (\text{B.10})$$

The right-hand side of equation (B.10) is equal to the product of $\log \lambda_1$ and the expected number of improvements in a time interval of length t . I can invoke the properties of the

Poisson distribution to argue that the expected number of improvements is $I_1 t$ (Feller 1968, p.159). Therefore,

$$\int_0^{\omega} \left[\sum_{m=0}^{\infty} f(m, t) \log \lambda_1^m \right] d\theta = I_1 t \log \lambda_1 \int_0^{\omega} d\theta = \omega I_1 t \log \lambda_1. \quad (\text{B.11})$$

Similarly, the second integral in the right-hand side of equation (A.9) is equal to the following:

$$\int_{\omega}^1 \log \lambda_0^j = \int_{\omega}^1 \left[\sum_{m=0}^{\infty} f(m, t) \log \lambda_0^m \right] d\theta = I_0 t \log \lambda_0 \int_{\omega}^1 d\theta = (1 - \omega) I_0 t \log \lambda_0. \quad (\text{B.12})$$

By substituting equations (B.11) and (B.12) into equation (B.9) I obtain the following:

$$\int_0^1 \log \lambda(\theta)^{x(\theta)} d\theta = \omega I_1 t \log \lambda_1 + (1 - \omega) I_0 \log \lambda_0. \quad (\text{B.13})$$

I calculate the second integral in equation (B.8) as follows:

$$\int_0^1 \log \lambda(\theta) d\theta = \int_0^{\omega} \log \lambda_1 d\theta + \int_{\omega}^1 \log \lambda_0 d\theta = \omega \log \lambda_1 + (1 - \omega) \log \lambda_0. \quad (\text{B.14})$$

Finally, substituting equations (B.13) and (B.14) into equation (B.8) I obtain the following:

$$\log u(t) = \log c(t) - \log \alpha_0 + \omega(I_1 t - 1) \log \lambda_1 + (1 - \omega)(I_0 t - 1) \log \lambda_0. \quad (\text{B.15})$$

The economy's long-run Schumpeterian growth is defined as the rate of growth of sub-utility $u(t)$, $g_u = \dot{u}(t)/u(t)$. By taking logs and differentiating equation (B.15) with respect to time and substituting equation (31) from the main text, I obtain the following:

$$g_u = \frac{\mu(c-1)}{k\alpha_R[\mu - \omega(\mu-1)]} + g_N - \rho + \dot{\omega}[(I_1 t - 1) \log \lambda_1 - (I_0 t - 1) \log \lambda_0] \\ + \omega(I_1 + t\dot{I}_1) \log \lambda_1 + (1 - \omega)(I_0 + t\dot{I}_0) \log \lambda_0. \quad (\text{B.16})$$

Evaluating equation (B.16) at the two steady-states, I obtain the following:

$$g_u = \begin{cases} I_0 \log \lambda_0 & \text{for } \omega = 0 \\ I_1 \log \lambda_1 & \text{for } \omega = 1. \end{cases} \quad (\text{B.17})$$

Equation (B.17) implies that the long-run Schumpeterian growth rate is higher at the final steady-state than at the initial steady-state.

By setting equation (31) equal to zero, I obtain per capita consumption expenditure as a function of the measure of industries that have adopted the new GPT as follows:

$$c(\omega) = \frac{(\rho - g_N)k\alpha_R[\mu - \omega(\mu - 1)]}{\mu} + 1. \quad (\text{B.18})$$

Evaluating equation (B.18) at the two steady-states, I obtain

$$\hat{c}(\omega) = \begin{cases} \hat{c}(0) = (\rho - g_N)k\alpha_R + 1 & \text{for } \omega = 0 \\ \hat{c}(1) = \frac{(\rho - g_N)k\alpha_R}{\mu} + 1 & \text{for } \omega = 1. \end{cases} \quad (\text{B.19})$$

Equation (B.19) implies that $\hat{c}(1) < \hat{c}(0)$.

At each steady-state, per capita consumption expenditure is constant. That is $\dot{\hat{c}} = 0$. Then equation (8) implies that the interest rate, $\hat{r}$, is equal to the subjective discount rate, ρ .

Equations (20) and (13) in the main text imply that

$$\frac{V(1)}{N} = \frac{V_1}{N} < \frac{V(0)}{N} = \frac{V_0}{N}. \quad (\text{B.20})$$

This completes the proof of Proposition 2.

B.2.3 Proposition 3

In order to prove that there exists locally a negative sloping saddle path, I use a polynomial of order one to linearize the nonlinear differential equations (1) and (31)

around their steady-state values ($\hat{c} = \frac{(\rho - g_N)k\alpha_R}{\mu} + 1$, $\hat{\omega} = 1$).

Equations (31) and (1) in the main text can be written as follows:

$$\begin{aligned}\dot{c} &= a_1 c + b_1 \omega \\ \dot{\omega} &= b_2 \omega,\end{aligned}\tag{B.21}$$

where $a_1 = \frac{[(\rho - g_N)k\alpha_R + \mu]}{k\alpha_R}$, $b_1 = \frac{[(\rho - g_N)k\alpha_R + \mu][(\rho - g_N)k\alpha_R(\mu - 1)]}{\mu k\alpha_R}$, and

$$b_2 = -\delta.$$

Suppose $c = Ae^n$ and $\bar{\omega} = Be^n$ are the particular solutions to the homogenous system (B.21). Substitution of these proposed solutions into system (B.18) yields the following:

$$\begin{aligned}rAe^n &= a_1 Ae^n + b_1 Be^n \\ rBe^n &= b_2 Be^n.\end{aligned}\tag{B.22}$$

Dividing by e^n throughout system (B.22), collecting terms, and rewriting in matrix notation, I obtain the following:

$$\begin{bmatrix} a_1 - r & b_1 \\ 0 & b_2 - r \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.\tag{B.23}$$

In order for the solution to (B.23) be other than $A = B = 0$, the coefficient matrix in (B.23) must be singular. That is,

$$\begin{vmatrix} a_1 - r & b_1 \\ 0 & b_2 - r \end{vmatrix} = 0. \quad (\text{B.24})$$

Expanding (B.24) yields a quadratic equation in r as follows:

$$r^2 - r(a_1 + b_2) + a_1 b_2 = 0. \quad (\text{B.25})$$

Computing the roots of the quadratic equation in (B.25) I obtain the following:

$$r_1 = \frac{(a_1 + b_2) + \sqrt{(a_1 + b_2)^2 - 4a_1 b_2}}{2}, \quad (\text{B.26})$$

$$r_2 = \frac{(a_1 + b_2) - \sqrt{(a_1 + b_2)^2 - 4a_1 b_2}}{2}.$$

Since $a_1 > 0$ and $b_2 < 0$, the roots are real. Because the argument of the square root function exceeds $(a_1 + b_2)^2$, the smaller of the roots is negative. The larger root is positive. Hence, the roots are real and of opposite sign; the stationary point is a saddlepoint.

In order to prove that there exists globally a negative sloping saddle path, I use the time elimination method. The slope of the policy function can be obtained by taking the ratio of the two differential equations that govern the dynamic behavior of the economy as follows:

$$\frac{\partial c}{\partial \omega} = \frac{\dot{c}}{\dot{\omega}} = \frac{\frac{[c^2(\omega)\mu - c(\omega)\mu]}{k\alpha_R[\mu - \omega(\mu - 1)]} + c(\omega)(g_N - \rho)}{\delta\omega - \delta\omega^2}. \quad (\text{B.27})$$

Time does not appear in the above equation. To solve this equation numerically, I need one boundary condition; that is, I have to know one point, (c, ω) , that lies on the stable

arm. Although I do not know the initial pair, $[c(0), \omega(0)]$, I know that the policy function goes through the steady state $(\hat{c}, \hat{\omega})$.

The slope of the policy function at the steady state is the following:

$$c'(\hat{\omega}) = \frac{\frac{\dot{c}}{c}}{\frac{\dot{\omega}}{\omega}} = \frac{0}{0}, \quad (\text{B.28})$$

which is an indeterminate form.

Applying the L'Hôpital's rule to equation (B.28) and evaluating it at the steady state values, I obtain the following:

$$c'(\hat{\omega}) = \frac{[1 - c(\hat{\omega})](\mu - 1)c(\hat{\omega})}{\{\mu[2c(\hat{\omega}) - 1] + (\delta + g_N - \rho)k\alpha_N\}}. \quad (\text{B.29})$$

From the phase diagram in Figure 3-2, I can sign the following expressions:

$[2c(\hat{\omega}) - 1] > 0$ and $[1 - c(\hat{\omega})] < 0$. If $\delta > (g_N - \rho)$, then the expression in (B.29) is negative. Thus, if the rate of diffusion is high enough (higher than the effective discount rate) the slope of the stable arm is negative.

Along the saddle path, the measure of industries that adopt the new GPT evolves according to equation (1). As ω increases, equation (B.29) implies that per capita consumption expenditure decreases. Along the saddle path, $\dot{c}/c < 0$, which implies that the market interest rate, r , is lower than the subjective interest rate, ρ (from equation (8) in the main text). Once the diffusion process is about to complete and the economy approaches the new steady state, the market interest rate increases to become equal with the constant subjective interest rate at the new steady state.

In order to analyze the behavior of the two innovation rates along the diffusion process, I differentiate equations (B.16) and (B.17) with respect to ω as follows:

$$\frac{\partial I_1}{\partial \omega} = \left(\frac{\mu(\lambda_1 - 1)}{k\alpha_R \lambda_1} \right) \frac{\partial c}{\partial \omega} - \frac{\partial \tau}{\partial \omega} < 0, \quad (\text{B.30})$$

$$\frac{\partial I_0}{\partial \omega} = \left(\frac{(\lambda_0 - 1)}{k\alpha_R \lambda_0} \right) \frac{\partial c}{\partial \omega} - \frac{\partial \tau}{\partial \omega} < 0, \quad (\text{B.31})$$

since $\partial c / \partial \omega < 0$ and $\partial \tau / \partial \omega > 0$.

By taking the difference between equations (B.30) and (B.31), I obtain the following:

$$\frac{\partial I_1}{\partial \omega} - \frac{\partial I_0}{\partial \omega} = \left(\frac{\mu(\lambda_1 - 1)}{k\alpha_R \lambda_1} - \frac{(\lambda_0 - 1)}{k\alpha_R \lambda_0} \right) \frac{\partial c}{\partial \omega} < 0. \quad (\text{B.32})$$

That is, along the diffusion process, the innovation rate for the industries that have adopted the new GPT decreases more than that of the industries that have not adopted the new GPT.

In terms of the growth rate, differentiating equation (B.12) with respect to time and taking the limit when the economy approaches the two steady-states, I obtain the following equations:

$$\lim_{\omega \rightarrow 0} \left[\frac{\partial g_u}{\partial \tau} \right] = \frac{\dot{c}}{k\alpha_R} + \tilde{I}_0 \log \lambda_0 + 2\dot{I}_0 \log \lambda_0 < 0, \quad (\text{B.33})$$

$$\lim_{\omega \rightarrow 1} \left[\frac{\partial g_u}{\partial \tau} \right] = \frac{\mu \dot{c}}{k\alpha_R} + \tilde{I}_1 \log \lambda_1 + 2\dot{I}_1 \log \lambda_1 < 0. \quad (\text{B.34})$$

Since the signs of equations (B.33) and (B.34) are negative, the growth rate decreases both in the initial stage and towards the final stage of the diffusion process. Since the quantity of equation (B.33) is smaller than that of equation (B.34) in absolute values, the growth rate decreases more towards the final stage than in the initial stage.

B.2.4 Proposition 4

The growth rate of the stock market is given by equation (B.2). From equation (B.2), it is obvious that the growth rate of the stock market depends on the rate of GPT diffusion process, δ , the magnitude of the GPT-ridden R&D productivity gains, μ , and the rate of population growth, g_N .

By differentiating equation (B.2) with respect to ω , I obtain the following:

$$\frac{\partial g_v}{\partial \omega} = \frac{\delta(1-\mu)(k\alpha_R)^2[(1-\omega)^2\mu - \omega^2]}{[\omega k\alpha_R + (1-\omega)k\alpha_R\mu]^2}. \quad (\text{B.35})$$

The sign of (B.35) depends on the sign of the expression $[(1-\omega)^2\mu - \omega^2]$. When $\omega < \sqrt{\mu}/(1+\sqrt{\mu})$, the growth rate of the aggregate stock value decreases. When $\omega > \sqrt{\mu}/(1+\sqrt{\mu})$, the growth rate of the aggregate stock value increases. That is, along the diffusion process, the growth rate of the aggregate stock value follows a U-shaped path.

Differentiating equation (B.35) with respect to the diffusion rate, δ , I obtain the following:

$$\frac{\partial g_v}{\partial \delta} = \frac{\omega(1-\omega)(1-\mu)}{[1 + (1-\omega)\mu]} < 0. \quad (\text{B.36})$$

Equation (B.36) implies that when the diffusion rate increases, the growth rate of the aggregate stock market decreases.

Differentiating equation (B.35) with respect to the diffusion rate, δ , I obtain the following:

$$\frac{\partial(\partial g_v / \partial \omega)}{\partial \delta} = \frac{(1-\mu)[(1-\omega)^2\mu - \omega^2]}{[\omega + (1-\omega)\mu]^2}. \quad (\text{B.37})$$

The sign of equation (B.37) depends on the sign of the expression $[(1-\omega)^2\mu - \omega^2]$.

When the rate of the diffusion process of the new GPT increases, the slope of the growth of the stock market increases (i.e., gets steeper) for $\omega < \sqrt{\mu}/(1+\sqrt{\mu})$ and decreases (i.e., gets for $\omega > \sqrt{\mu}/(1+\sqrt{\mu})$).

By differentiating equation (B.2) with respect to μ , I obtain the following:

$$\frac{\partial g_v}{\partial \mu} = -\frac{\omega\delta(1-\omega)}{[\omega + (1-\omega)\mu]^2} < 0. \quad (\text{B.38})$$

The sign of equation (B.38) implies that the growth rate of the stock market decreases when the productivity gains from the new GPT are larger.

By differentiating equation (B.2) with respect to μ , I obtain the following:

$$\frac{\partial g_v}{\partial g_n} = 1 > 0. \quad (\text{B.39})$$

The sign of equation (B.39) implies that the growth rate of the stock market increases at the rate of the population growth.

B.2.5 Proposition 5

The aggregate investment is given by equation (B.14). Differentiating equation (B.14) with respect to the measure of industries that adopt the new GPT, I obtain the following:

$$\frac{\partial I}{\partial \omega} = \frac{\partial I_e}{\partial \omega}(1-\omega) + \frac{\partial I_1}{\partial \omega}\omega + (I_1 - I_e). \quad (\text{B.40})$$

The first two terms in equation (B.40) are negative (from equations (B.30) and (B.31)). The third term is positive. Thus, the aggregate investment will decrease or increase during the diffusion process depending on the magnitude of these signs.

The first two terms are the slopes of the solid curves depicted in Figure 3-7. These slopes depend on the rate of diffusion of the new GPT (see equations (B.27), (B.30), and (B.31)). On contrast, the third term of equation (B.40) does not depend on the rate of diffusion of the new GPT. Thus, when the rate of diffusion of the new GPT increases and more industries switch to the new GPT faster, the negative effect dominates the positive effect and the aggregate investment decreases.

$$\lim_{\alpha \rightarrow 1} I_1(\omega) = \frac{\mu(\lambda_1 - 1)}{k\alpha_R \lambda_1} c(\omega) + g_N - \rho. \quad (\text{B.41})$$

$$\lim_{\alpha \rightarrow 1} I_0(\omega) = \frac{(\lambda_0 - 1)}{k\alpha_R \lambda_0} c(\omega) + g_N - \rho. \quad (\text{B.42})$$

Equations (B.41) and (B.42) imply that when the measure of industries that adopt the new GPT approaches the final steady-state, the innovation rates of the industries that have adopted the new GPT and the innovation rate of industries that have not adopted the new GPT are not the same. That implies that the innovation rate of the industries that have not adopted the new GPT jumps upward (i.e., when the last industry switches to the new GPT, I_0 becomes I_1).

APPENDIX C
PROOFS OF PROPOSITIONS IN CHAPTER 4

C.1 Proof of Lemma 1

C.1.1 Lemma 1

Lemma 1 results from equations (20) and (22) (after taking into account equations (12) and (13)). Suppose that in industry $\theta' \in [0,1]$, R&D is undertaken by Home. Then, from the zero profit conditions, one can obtain the following inequality:

$$V^f(\theta') < V^h(\theta'). \quad (\text{C.1})$$

After substitution of the profit functions (equations (12) and (13)) into equation (20), the above inequality yields the following:

$$\omega < \frac{\alpha_Q^f(\theta')}{\alpha_Q^h(\theta')}. \quad (\text{C.2})$$

The reverse is true for any industry $\theta'' \in [0,1]$ that Foreign undertakes R&D as follows:

$$\omega > \frac{\alpha_Q^f(\theta'')}{\alpha_Q^h(\theta'')}. \quad (\text{C.3})$$

There is an industry $\tilde{\theta} \in [0,1]$, where both countries, Home and Foreign, undertake R&D, and for this industry, the zero profit conditions yield the following:

$$V^f(\tilde{\theta}) = V^h(\tilde{\theta}). \quad (\text{C.4})$$

The last equality yields

$$\omega = \frac{\alpha_Q^f(\tilde{\theta})}{\alpha_Q^h(\tilde{\theta})}. \quad (\text{C.5})$$

Since the ratio of relative productivities declines with the industry index (assumption (A.1)), in any industry $\theta \in [0, \tilde{\theta})$, Home has zero profits, while Foreign attains negative profits. Thus, for any industry that belongs to this interval, only Home conducts R&D. In any industry $\theta \in (\tilde{\theta}, 1]$, Foreign has zero profits, while Home attains negative profits. As a result, for any industry that belongs to this interval, only Foreign conducts R&D.

C.2 Proofs of Propositions 1, 2, 3, and 4 Under the PEG Specification

C.2.1 Proposition 1

First, I derive the mutual resource schedule (equation (35)) for the PEG specification. I solve for the integral of the left hand side in equation (28) and substitute it into equation (33) and obtain the following:

$$\bar{N}^r(t) = \frac{\bar{E}(t)\tilde{\theta}}{\omega} - \tilde{\theta}k\alpha_R(\rho - g_N). \quad (C.6)$$

Then, from equation (29), I solve for I and substitute it into equation (34) and obtain the following:

$$\bar{E}(t) = \frac{\bar{N}^r(t)}{(1 - \tilde{\theta})} + k\alpha_R(\rho - g_N). \quad (C.7)$$

Substituting equation (C.7) into equation (C.6), I obtain the following mutual resource condition (equation (29) in Chapter 3):

$$RS(\tilde{\theta}) = \omega = \frac{\left[\frac{\bar{N}^r(t)}{(1 - \tilde{\theta})} + k\alpha_R(\rho - g_N) \right] \tilde{\theta}}{\bar{N}^b(t) + \tilde{\theta}k\alpha_R(\rho - g_N)}. \quad (C.8)$$

For the expressions and calculations that follow, I use the letter M^1 for the numerator of equation (C.8) and the letter M^2 for its denominator.

$$\frac{d\omega}{d\tilde{\theta}} = \frac{\left[\frac{\bar{N}^f(t)}{(1-\tilde{\theta})} + k\alpha_k(\rho - g_N) + \tilde{\theta} \frac{\bar{N}^f(t)}{(1-\tilde{\theta})^2} \right] M^2 - M^1 \alpha_k k(\rho - g_N)}{(M^2)^2}. \quad (C.9)$$

Expanding the numerator of equation (C.9), I obtain the following expression, which is positive:

$$\left[\frac{\bar{N}^f(t)}{(1-\tilde{\theta})} + k\alpha_k(\rho - g_N) \right] \bar{N}^h(t) + \frac{\tilde{\theta} \bar{N}^f(t)}{(1-\tilde{\theta})^2} [\bar{N}^f(t) + \tilde{\theta} k\alpha_k(\rho - g_N)] > 0. \quad (C.10)$$

This implies that (C.9) is also positive. Therefore, the relative wage, ω , increases in $\tilde{\theta}$.

First, I will prove equation (10) that is assumed in Proposition 1.

$$\frac{\partial RS(\tilde{\theta})}{\partial \bar{N}^h(t)} = - \frac{\left[\frac{\bar{N}^f(t)}{(1-\tilde{\theta})} + k\alpha_k(\rho - g_N) \right] \tilde{\theta}}{[\bar{N}^h(t) + \tilde{\theta} k\alpha_k(\rho - g_N)]^2} < 0, \quad (C.11)$$

$$\frac{\partial RS(\tilde{\theta})}{\partial \bar{N}^f(t)} = - \frac{\frac{\tilde{\theta}}{(1-\tilde{\theta})}}{[\bar{N}^h(t) + \tilde{\theta} k\alpha_k(\rho - g_N)]} > 0, \quad (C.12)$$

I also calculate the following derivations:

$$\frac{\partial RS(\tilde{\theta})}{\partial k} = - \frac{\tilde{\theta} \alpha_k(\rho - g_N) [M^2 - M^1]}{[\bar{N}^h(t) + \tilde{\theta} k\alpha_k(\rho - g_N)]^2} \succcurlyeq ? \quad (C.13)$$

Expanding the expression $[M^2 - M^1]$ in equation (C.13), I obtain the following:

$$[M^2 - M^1] = [\bar{N}^h(t) - \frac{\tilde{\theta}}{(1-\tilde{\theta})} \bar{N}^f(t)] \succcurlyeq ?$$

Thus, if the foreign country's relative size is high enough, the expression above is negative. I make the following assumption to examine the comparative statics:

$$\frac{\bar{N}^f}{\bar{N}^h} > \frac{(1-\tilde{\theta})}{\tilde{\theta}}. \quad (C.14)$$

Assumption (C.14) implies that $[M^2 - M^1] < 0$ and the sign of equation (C.13) is negative.

$$\frac{\partial RS(\tilde{\theta})}{\partial \alpha_R} = \frac{\tilde{\theta} k (\rho - g_N) [M^2 - M^1]}{[N^h(t) + \tilde{\theta} k \alpha_R (\rho - g_N)]^2} < 0, \quad (C.15)$$

$$\frac{\partial RS(\tilde{\theta})}{\partial \rho} = \frac{\tilde{\theta} k \alpha_R [M^2 - M^1]}{[N^h(t) + \tilde{\theta} k \alpha_R (\rho - g_N)]^2} < 0, \quad (C.16)$$

$$\frac{\partial RS(\tilde{\theta})}{\partial g_N} = - \frac{\tilde{\theta} k \alpha_R [M^2 - M^1]}{[N^h(t) + \tilde{\theta} k \alpha_R (\rho - g_N)]^2} > 0. \quad (C.17)$$

Thus, for sufficiently large $\frac{\bar{N}^f}{N^h}$, the RS schedule defined by equation (C.8)

intersects the RD schedule such that a unique steady-state equilibrium exists and $\omega > 1$.

The intersection of the downward sloping $RD(\tilde{\theta})$ schedule and the upward sloping

$MR(\tilde{\theta})$ schedule at point E determines the equilibrium relative wage, ω , and the

marginal industry $\tilde{\theta}$ in which both countries undertake production in goods and R&D services.

C.2.2 Proposition 2

I can write the two equilibrium relationships governing Figure 1 in a more general form as follows:

$$\omega = RD(\tilde{\theta}, \alpha_Q^h, \alpha_Q^f), \quad D_1 < 0, D_2 < 0, D_3 > 0 \quad (C.18)$$

$$\omega = MR(\tilde{\theta}, \bar{N}^f, \bar{N}^f, k, \alpha_R, \rho, g_N), \quad R_1 > 0, R_2 > 0, R_3 < 0, \quad (C.19)$$

$$R_4 > < 0?, R_5 > < 0?, R_6 > < 0?, R_7 > < 0?$$

I totally differentiate equations (C.18) and (C.19) and obtain the following system of two equations in the differentials of two endogenous variables as follows:

$$d\omega - D_1 d\tilde{\theta} = D_2 d\alpha_Q^h + D_3 d\alpha_Q^f$$

$$d\omega - R_1 d\tilde{\theta} = R_2 d\bar{N}^f + R_3 d\bar{N}^h + R_4 dk + R_5 d\alpha_R + R_6 d\rho + R_7 dg_N \quad (\text{C.20})$$

I can write the system (C.20) in the reduced form as follows:

$$\begin{bmatrix} 1 & -D_1 \\ 1 & -R_1 \end{bmatrix} \begin{bmatrix} d\omega \\ d\tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & D_2 & D_3 \\ R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\bar{N}^f \\ d\bar{N}^h \\ dk \\ d\alpha_R \\ d\rho \\ dg_N \\ d\alpha_Q^h \\ d\alpha_Q^f \end{bmatrix}, \quad (\text{C.21})$$

Assumption (C.14) implies that $R_4 < 0$, $R_5 < 0$, $R_6 < 0$, and $R_7 > 0$.

I calculate the determinant of the matrix of the endogenous variables (which I denote with Δ) as follows:

$$\Delta = \begin{vmatrix} 1 & -D_1 \\ 1 & -R_1 \end{vmatrix} = D_1 - R_1 < 0. \quad (\text{C.22})$$

Using the system of equations given by (C.21), I establish the comparative steady-state results for the PEG specification regarding ω . I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (C.21) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\omega}{d\bar{N}^f} = \frac{D_1 R_2}{\Delta} > 0, \quad (\text{C.23})$$

$$\frac{d\omega}{d\bar{N}^h} = \frac{D_1 R_3}{\Delta} < 0, \quad (\text{C.24})$$

$$\frac{d\omega}{dk} = \frac{D_1 R_4}{\Delta} < 0, \quad (\text{C.25})$$

$$\frac{d\omega}{d\alpha_R} = \frac{D_1 R_5}{\Delta} < 0, \quad (C.26)$$

$$\frac{d\omega}{d\rho} = \frac{D_1 R_6}{\Delta} < 0, \quad (C.27)$$

$$\frac{d\omega}{dg_N} = \frac{D_1 R_7}{\Delta} > 0, \quad (C.28)$$

$$\frac{d\omega}{d\alpha_Q^h} = \frac{-D_2 R_1}{\Delta} < 0, \quad (C.29)$$

$$\frac{d\omega}{d\alpha_Q^f} = \frac{-D_3 R_1}{\Delta} > 0. \quad (C.30)$$

The signs of equations (C.23) through (C.30) prove Proposition 2.

C.2.3 Proposition 3

Using the system of equations given by (C.21), I establish the comparative steady-state results for the PEG specification regarding $\tilde{\theta}$. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (C.21) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\tilde{\theta}}{dN^f} = \frac{R_2}{\Delta} < 0, \quad (C.31)$$

$$\frac{d\tilde{\theta}}{dN^h} = \frac{R_3}{\Delta} > 0, \quad (C.32)$$

$$\frac{d\tilde{\theta}}{dk} = \frac{R_4}{\Delta} > 0, \quad (C.33)$$

$$\frac{d\tilde{\theta}}{d\alpha_R} = \frac{R_5}{\Delta} > 0, \quad (C.34)$$

$$\frac{d\tilde{\theta}}{d\rho} = \frac{R_6}{\Delta} > 0, \quad (C.35)$$

$$\frac{d\tilde{\theta}}{dg_N} = \frac{R_7}{\Delta} < 0, \quad (C.36)$$

$$\frac{d\tilde{\theta}}{d\alpha_Q^h} = -\frac{D_2}{\Delta} < 0, \quad (C.37)$$

$$\frac{d\tilde{\theta}}{d\alpha_Q^f} = -\frac{D_3}{\Delta} > 0. \quad (C.38)$$

The signs of equations (C.31) through (C.38) prove Proposition 3.

C.2.4 Proposition 4

For the calculations in this section, I use the following notation:

$$S = (\rho + I - g_N) \text{ and } A(\tilde{\theta}) = \int_0^{\tilde{\theta}} \frac{\alpha_Q^h(\theta)}{\alpha_Q^f(\theta)} d\theta.$$

I totally differentiate the system of equations (28), (29), (33), and (34), and obtain the following:

$$\frac{A(\tilde{\theta})}{\lambda} d\bar{E} + k\alpha_R \tilde{\theta} dI + \left[\frac{\bar{E}\alpha_Q^h(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right] d\tilde{\theta} = d\bar{N}^h + \frac{\bar{E}A(\tilde{\theta})}{\lambda^2} d\lambda - \tilde{\theta} k I d\alpha_R - \alpha_R \tilde{\theta} I dk, \quad (C.39)$$

$$\frac{(1-\tilde{\theta})}{\lambda} d\bar{E} - \left(\frac{\bar{E}}{\lambda} + I k \alpha_R \right) d\tilde{\theta} + k\alpha_R (I - \tilde{\theta}) dI \quad (C.40)$$

$$= d\bar{N}^f + \frac{(I-\tilde{\theta})\bar{E}}{\lambda^2} d\lambda - I k (1-\tilde{\theta}) d\alpha_R - I \alpha_R (1-\tilde{\theta}) dk$$

$$\left(\tilde{\theta} - \frac{\omega A(\tilde{\theta})}{\lambda} \right) d\bar{E} + \left[\bar{E} - \frac{\omega \bar{E} \alpha_Q^h(\tilde{\theta})}{\lambda \alpha_Q^f(\tilde{\theta})} - \omega \alpha_R k S \right] d\tilde{\theta} - \left(\frac{\bar{E} A(\tilde{\theta})}{\lambda} + \tilde{\theta} \alpha_R k S \right) d\omega - \tilde{\theta} \omega k \alpha_R dI \quad (C.41)$$

$$= \tilde{\theta} \omega k S d\alpha_R - \frac{\bar{E} \omega A(\tilde{\theta})}{\lambda} d\lambda + \tilde{\theta} \omega \alpha_R S dk + \tilde{\theta} \omega k \alpha_R d\rho - \tilde{\theta} \omega k \alpha_R dg_N$$

$$\frac{(\lambda-1)}{\lambda} d\bar{E} - k\alpha_R dI = \alpha_R S dk + kS d\alpha_R + k\alpha_R d\rho - k\alpha_R dg_N - \frac{\bar{E}}{\lambda^2} d\lambda. \quad (C.42)$$

I can rewrite the system of differential equations (C.39)- (C.42) in the following reduced form:

$$\begin{bmatrix} \frac{A(\tilde{\theta})}{\lambda} & k\alpha_R \tilde{\theta} & 0 & \left(\frac{\bar{E}\alpha_Q^h(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \\ \frac{(1-\tilde{\theta})}{\lambda} & k\alpha_R (1-\tilde{\theta}) & 0 & -\left(\frac{\bar{E}}{\lambda} + k\alpha_R I \right) \\ \left(\tilde{\theta} - \frac{\omega A(\tilde{\theta})}{\lambda} \right) & -\tilde{\theta}\omega k\alpha_R & -\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) & \left(\bar{E} - \frac{\omega\bar{E}\alpha_Q^h(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} - \omega k\alpha_R S \right) \\ \frac{(\lambda-1)}{\lambda} & -k\alpha_R & 0 & 0 \end{bmatrix} \begin{bmatrix} d\tilde{E} \\ dI \\ d\omega \\ d\tilde{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{\bar{E}A(\tilde{\theta})}{\lambda^2} & -\alpha_R \tilde{\theta} I & -\tilde{\theta} k I & 0 & 0 \\ 0 & 1 & \frac{(1-\tilde{\theta})\bar{E}}{\lambda^2} & -I\alpha_R (1-\tilde{\theta}) & -k(1-\tilde{\theta}) I & 0 & 0 \\ 0 & 0 & -\frac{\bar{E}A(\tilde{\theta})\omega}{\lambda^2} & \tilde{\theta}\omega\alpha_R S & \tilde{\theta}\omega k S & \tilde{\theta}\omega\alpha_R k & -\tilde{\theta}\omega\alpha_R k \\ 0 & 0 & -\frac{\bar{E}}{\lambda^2} & \alpha_R S & k S & k\alpha_R & -k\alpha_R \end{bmatrix} \begin{bmatrix} d\bar{N}^f \\ d\bar{N}^h \\ d\lambda \\ dk \\ d\alpha_R \\ d\rho \\ dg_N \end{bmatrix} \quad (C.43)$$

I calculate the determinant of the matrix of the endogenous variables (which I denote with Δ_2) as follows:

$$\Delta_2 = \begin{vmatrix} \frac{A(\tilde{\theta})}{\lambda} & k\alpha_R \tilde{\theta} & 0 & \left(\frac{\bar{E}\alpha_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \\ \frac{(1-\tilde{\theta})}{\lambda} & k\alpha_R (1-\tilde{\theta}) & 0 & -\left(\frac{\bar{E}}{\lambda} + k\alpha_R I \right) \\ \left(\tilde{\theta} - \frac{\omega A(\tilde{\theta})}{\lambda} \right) & -\tilde{\theta}\omega k\alpha_R & -\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) & \left(\bar{E} - \frac{\omega\bar{E}\alpha_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} - \omega k\alpha_R S \right) \\ \frac{(\lambda-1)}{\lambda} & -k\alpha_R & 0 & 0 \end{vmatrix} =$$

$$-\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \begin{vmatrix} \frac{A(\tilde{\theta})}{\lambda} & k\alpha_R \tilde{\theta} & \left(\frac{\bar{E}\alpha_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \\ \frac{(1-\tilde{\theta})}{\lambda} & k\alpha_R (1-\tilde{\theta}) & -\left(\frac{\bar{E}}{\lambda} + k\alpha_R I \right) \\ \frac{(\lambda-1)}{\lambda} & -k\alpha_R & 0 \end{vmatrix}$$

$$= \left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left[\frac{(\lambda-1)}{\lambda} \left[k\alpha_R \tilde{\theta} \left(\frac{\bar{E}}{\lambda} - k\alpha_R \right) + k\alpha_R I (1-\tilde{\theta}) \right] \left(\frac{\bar{E}\alpha_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \right]$$

$$+ k\alpha_R \left[\frac{A(\tilde{\theta})}{\lambda} \left(\frac{\bar{E}}{\lambda} + k\alpha_R \right) + \frac{(1-\tilde{\theta})}{\lambda} \left(\frac{\bar{E}\alpha_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \right]$$

The above expression is referred to as (C.44) and is positive. Thus, $\Delta_2 > 0$.

Using the system of equations given by (C.43), I establish the comparative steady-state results for the PEG specification regarding I. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (C.43) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{dI}{dN^f} = \frac{\frac{(\lambda-1)}{\lambda} \left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left(\frac{\bar{E}a_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right)}{\Delta_2} > 0, \quad (C.45)$$

$$\frac{dI}{dN^h} = \frac{\frac{(\lambda-1)}{\lambda} \left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left(\frac{\bar{E}}{\lambda} - I k\alpha_R \right)}{\Delta_2} > 0, \quad (C.46)$$

$$\frac{dI}{d\alpha} = \frac{\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left[\frac{\bar{E}A(\tilde{\theta})}{\lambda^2} \left(\frac{\bar{E}}{\lambda} + k\alpha_R I \right) + \frac{\bar{E}(1-\tilde{\theta})}{\lambda^2} \left(\frac{\bar{E}a_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \right]}{\Delta_2} > 0, \quad (C.47)$$

$$\frac{dI}{dk} = - \frac{\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left[\left(\frac{(\lambda-1)\alpha_R \bar{I}\tilde{\theta} + \alpha_R S A}{\lambda} \right) \left(\frac{\bar{E}}{\lambda} + k\alpha_R I \right) \right]}{\Delta_2} < 0, \quad (C.48)$$

$$+ \frac{\left(\frac{(\lambda-1)(1-\tilde{\theta})I\alpha_R + \alpha_R S(1-\tilde{\theta})}{\lambda} \right) \left(\frac{\bar{E}a_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right)}{\Delta_2}$$

$$\frac{dI}{d\alpha_R} = - \frac{\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left(\frac{(\lambda-1)}{\lambda} kI\tilde{\theta} + kS \frac{A(\tilde{\theta})}{\lambda} \right) \left(\frac{\bar{E}}{\lambda} + I k\alpha_R \right)}{\Delta_2} < 0, \quad (C.49)$$

$$+ \frac{\left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left(\frac{(\lambda-1)}{\lambda} kI(1-\tilde{\theta}) + kS \frac{(1-\tilde{\theta})}{\lambda} \right) \left(\frac{\bar{E}a_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right)}{\Delta_2}$$

$$\frac{dI}{dp} = - \frac{k\alpha_R \left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left[\frac{A(\tilde{\theta})}{\lambda} \left(\frac{\bar{E}}{\lambda} + I k\alpha_R \right) + \frac{(1-\tilde{\theta})}{\lambda} \left(\frac{\bar{E}a_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \right]}{\Delta_2} < 0, \quad (C.50)$$

$$\frac{dI}{dg_N} = \frac{k\alpha_R \left(\frac{\bar{E}A(\tilde{\theta})}{\lambda} + \tilde{\theta}k\alpha_R S \right) \left[\frac{A(\tilde{\theta})}{\lambda} \left(\frac{\bar{E}}{\lambda} + I k\alpha_R \right) + \frac{(1-\tilde{\theta})}{\lambda} \left(\frac{\bar{E}a_Q^b(\tilde{\theta})}{\lambda\alpha_Q^f(\tilde{\theta})} + k\alpha_R I \right) \right]}{\Delta_2} > 0. \quad (C.51)$$

C.3 Proofs of Propositions 5, 6, and 7 Under the TEG Specification

C.3.1 Proposition 5

First, I will derive the RD schedule under the TEG specification (equation (23)).

Evaluating equations (38) and (39) at $\tilde{\theta}$, I obtain the following:

$$\frac{\bar{E}(t)[1 - \frac{\omega \alpha_Q^h(\tilde{\theta})}{\lambda \alpha_Q^f(\tilde{\theta})}]}{[\rho + I(\tilde{\theta}) - g_N]} = \frac{\bar{E}(t)(\lambda - 1)}{[\rho + I(\tilde{\theta}) - g_N]}. \quad (C.52)$$

Solving equation (C.52) for ω yields the following equation (equation (23) in the main text):

$$\omega = \frac{\alpha_Q^f(\tilde{\theta})}{\alpha_Q^h(\tilde{\theta})}. \quad (C.53)$$

Next, I derive the mutual resource schedule (equation (46)) under the TEG specification.

First, I solve for $\bar{X}$ in equation (41) and substitute it into equation (44) and obtain the following:

$$\bar{N}^h(t) = \frac{\bar{E}(t)A}{\lambda} - \frac{g_N}{\mu} \left[-\frac{\bar{E}(\tilde{\theta}\lambda - \omega A)}{\omega\lambda(\rho + \frac{g_N}{\mu} - g_N)} \right]. \quad (C.54)$$

Then, from equation (42), I solve for $\bar{X}$ and substitute it into equation (45) and obtain the following:

$$\bar{E}(t) = \frac{\bar{N}^f \mu \lambda S}{(1 - \tilde{\theta})[\mu S + g_N(\lambda - 1)]}, \quad (C.55)$$

where $S = (\rho + \frac{g_N}{\mu} - g_N)$.

Substituting equation (C.55) into equation (C.54), I obtain the following mutual resource schedule (equation (46)):

$$\omega = RS(\tilde{\theta}) = \frac{\bar{N}^f \lambda g_N \tilde{\theta}}{\bar{N}^b (1 - \tilde{\theta}) [\mu(\rho + \frac{g_N}{\mu} - g_N) + (\lambda - 1)g_N] - \bar{N}^f [\mu(\rho + \frac{g_N}{\mu} - g_N) - g_N] A(\tilde{\theta})}, \quad (C.56)$$

$$\text{where } A(\tilde{\theta}) = \int_0^{\tilde{\theta}} \frac{\alpha_Q^b(\theta)}{\alpha_Q^f(\theta)} d\theta.$$

For the expressions and calculations that follow, I use the letter M^2 for the denominator of equation (C.56).

$$\frac{d\omega}{d\tilde{\theta}} = \frac{\bar{N}^f \lambda g_N M^2 + \bar{N}^f \lambda g_N \tilde{\theta} \bar{N}^b \left[\mu S + (\lambda - 1)g_N + \frac{\alpha_Q^b(\tilde{\theta}) \bar{N}^f (\mu S - g_N)}{\alpha_Q^f(\tilde{\theta})} \right]}{(M^2)^2} > 0, \quad (C.57)$$

$$\frac{d\omega}{d\bar{N}^f} = \frac{\lambda g_N \tilde{\theta} M^2 + \bar{N}^f \lambda g_N \tilde{\theta} A(\tilde{\theta}) (\mu S - g_N)}{(M^2)^2} > 0, \quad (C.58)$$

$$\frac{d\omega}{d\bar{N}^b} = \frac{-\bar{N}^f \lambda g_N \tilde{\theta} \left\{ (1 - \tilde{\theta}) [\mu S + (\lambda - 1)g_N] \right\}}{(M^2)^2} < 0, \quad (C.59)$$

$$\frac{d\omega}{d\mu} = \frac{-\bar{N}^f \lambda g_N \tilde{\theta} \left\{ (\rho - g_N) \bar{N}^b (1 - \tilde{\theta}) - (\rho - g_N) \bar{N}^f A(\tilde{\theta}) \right\}}{(M^2)^2} > 0. \quad (C.60)$$

I assume that:

$$\bar{N}^f / \bar{N}^b > (1 - \tilde{\theta}) / A(\tilde{\theta}). \quad (C.61)$$

Assumption (C.61) allows me to sign the following two derivatives:

$$\frac{d\omega}{d\lambda} = \frac{\bar{N}^f g_N \tilde{\theta} M^2 - \bar{N}^f \lambda g_N \tilde{\theta} \bar{N}^b (1 - \tilde{\theta}) g_N}{(M^2)^2} < 0, \quad (C.62)$$

$$\frac{d\omega}{dg_N} = \frac{\bar{N}^f \lambda \tilde{\theta} [M^2 - g_N (\lambda - \mu) \bar{N}^h (1 - \tilde{\theta}) + \bar{N}^f A(\tilde{\theta}) \mu]}{(M^2)^2} < 0, \quad (C.63)$$

$$\frac{d\omega}{d\rho} = \frac{-\bar{N}^f \lambda g_N \tilde{\theta} \mu [\bar{N}^h (1 - \tilde{\theta}) - \bar{N}^f A(\tilde{\theta})]}{(M^2)^2} > 0, \quad (C.64)$$

Thus, for sufficiently large $\frac{\bar{N}^f}{\bar{N}^h}$, the RS schedule defined by equation (C.56)

intersects the RD schedule such that a unique steady-state equilibrium exists and $\omega > 1$.

The intersection of the downward sloping RD($\tilde{\theta}$) schedule and the upward sloping

RS($\tilde{\theta}$) schedule at point E determines the equilibrium relative wage, ω , and the marginal

industry $\tilde{\theta}$ in which both countries undertake production in goods and R&D services

C.3.2 Proposition 6

I can write the two equilibrium relationships governing Figure 9 in a more general form as follows:

$$\omega = RD(\tilde{\theta}, \alpha_Q^h, \alpha_Q^f), \quad D_1 < 0, D_2 < 0, D_3 > 0, \quad (C.65)$$

$$\omega = MR(\tilde{\theta}, \bar{N}^f, \bar{N}^h, \mu, \lambda, \rho, g_N), \quad R_1 > 0, R_2 > 0, R_3 < 0,$$

$$R_4 > 0?, R_5 > 0?, R_6 < 0?, R_7 > 0? \quad (C.66)$$

I totally differentiate equations (C.65) and (C.66) and obtain the following system of two equations in the differentials of two endogenous variables:

$$d\omega - D_1 d\tilde{\theta} = D_2 d\alpha_Q^h + D_3 d\alpha_Q^f,$$

$$d\omega - R_1 d\tilde{\theta} = R_2 d\bar{N}^f + R_3 d\bar{N}^h + R_4 d\lambda + R_5 d\mu + R_6 d\rho + R_7 dg_N. \quad (C.67)$$

I can write the system (C.67) in the reduced form as follows:

$$\begin{bmatrix} 1 & -D_1 \\ 1 & -R_1 \end{bmatrix} \begin{bmatrix} d\omega \\ d\tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & D_2 & D_3 \\ R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\bar{N}^r \\ d\bar{N}^h \\ d\mu \\ d\lambda \\ d\rho \\ dg_N \\ d\alpha_Q^h \\ d\alpha_Q^r \end{bmatrix}, \quad (\text{C.68})$$

Assumption (C.61) implies that $R_4 > 0$, $R_5 > 0$ and $R_6 < 0$.

I calculate the determinant of the matrix of the endogenous variables (which I denote with Δ_3) as follows:

$$\Delta_3 = \begin{vmatrix} 1 & -D_1 \\ 1 & -R_1 \end{vmatrix} = D_1 - R_1 < 0. \quad (\text{C.69})$$

Thus, I can calculate and sign the following derivatives:

$$\frac{d\omega}{d\bar{N}^r} = \frac{D_1 R_2}{\Delta} > 0, \quad (\text{C.70})$$

$$\frac{d\omega}{d\bar{N}^h} = \frac{D_1 R_3}{\Delta} < 0, \quad (\text{C.71})$$

$$\frac{d\omega}{d\mu} = \frac{D_1 R_4}{\Delta} > 0, \quad (\text{C.72})$$

$$\frac{d\omega}{d\rho} = \frac{D_1 R_5}{\Delta} > 0, \quad (\text{C.73})$$

$$\frac{d\omega}{dg_N} = \frac{D_1 R_6}{\Delta} > 0, \quad (\text{C.74})$$

$$\frac{d\omega}{d\alpha_Q^h} = -\frac{R_1 D_2}{\Delta} < 0, \quad (\text{C.75})$$

$$\frac{d\omega}{da_Q^r} = -\frac{R_1 D_3}{\Delta} > 0. \quad (C.76)$$

The signs of equations (C.70) through (C.76) prove Proposition 6.

C.3.3 Proposition 7

In order to calculate the effects on $\tilde{\theta}$, I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables with the corresponding column vector of the variable in consideration.

$$\frac{d\tilde{\theta}}{dN^r} = \frac{R_2}{\Delta} < 0, \quad (C.77)$$

$$\frac{d\tilde{\theta}}{dN^h} = \frac{R_3}{\Delta} > 0, \quad (C.78)$$

$$\frac{d\tilde{\theta}}{d\mu} = \frac{R_4}{\Delta} > 0, \quad (C.79)$$

$$\frac{d\tilde{\theta}}{d\rho} = \frac{R_5}{\Delta} < 0, \quad (C.80)$$

$$\frac{d\tilde{\theta}}{dg_N} = \frac{R_6}{\Delta} > 0, \quad (C.81)$$

$$\frac{d\tilde{\theta}}{da_Q^h} = \frac{-D_2}{\Delta} < 0, \quad (C.82)$$

$$\frac{d\tilde{\theta}}{da_Q^r} = \frac{-D_3}{\Delta} > 0. \quad (C.83)$$

The signs of equations (C.77) through (C.83) prove Proposition 7.

REFERENCES

- ABERNATHY, W. J. and UTTERBACK, J. M. (1978), "Patterns of Industrial Innovation", *Technology Review*, **80**, 41-47.
- AGHION, P. and HOWITT P. (1998a) *Endogenous Growth Theory* (Cambridge: MIT Press).
- AGHION, P. and HOWITT P. (1998b), "On the Macroeconomic Effects of Major Technological Change", in E. Helpman (eds.), *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press).
- ANDERSEN, B. (1999), "The Hunt for S-Shaped Growth Paths in Technological Innovation: A Patent Study", *Journal of Evolutionary Economics*, **9**, 487-526.
- ARROYO, C., DINOPOULOS, E., and DONALD, S. (1995), "Schumpeterian Growth and Capital Accumulation: Theory and Evidence" (Manuscript, University of Florida).
- BALDWIN, W. and SCOTT, J. (1987) *Market Structure and Technological Change* (Chur, Switzerland: Harwood Academic Publishers).
- BARRO, R. and SALA-I-MARTIN, X. (1995) *Economic Growth* (New York: McGraw-Hill).
- BRESNAHAN, T. and TRAJTENBERG, M. (1995), "General Purpose Technologies: Engines of Growth?", *Journal of Econometrics*, **65**, 83-108.
- CHENG, L. and DINOPOULOS, E. (1996), "A Multisectoral General Equilibrium Model of Schumpeterian Growth and Fluctuations", *Journal of Economic Dynamics & Control*, **20**, 905-923.
- COHEN, W. and KLEPPER, S. (1996), "Firm Size and the Nature of Innovation within Industries: The Case of Process and Product R&D," *The Review of Economics and Statistics* **78**, 232-243.
- DAVID, P. (1990), "The Dynamo and the Computer: A Historical Perspective on the Modern Productivity Paradox", *American Economic Review*, **80**, 355-61.
- DAVID, P. (1991), Computer and Dynamo. The Modern Productivity Paradox in a Not-Too-Distant Mirror. in *Technology and Productivity: The Challenge for Economic Policy*, Paris, OECD, 315-347.

- DEVINE, W. (1983), "From Shafts to Wires: Historical Perspective on Electrification", *Journal of Economic History*, **43**, 347-372.
- DINOPOULOS, E. (1994), "Schumpeterian Growth Theory: An Overview", *Osaka City University Economic Review*, **29**, 1-21.
- DINOPOULOS, E. and SEGERSTROM, P. (1999) "A Schumpeterian Model of Protection and Relative Wages", *American Economic Review* **89**, 450-472.
- DINOPOULOS, E. and SYROPOULOS, C. (2000), "Innovation and Rent Protection in the Theory of Schumpeterian Growth" (Manuscript, University of Florida).
- DINOPOULOS, E. and SYROPOULOS, C. (2001), "Globalization and Scale-Invariant Growth", (University of Florida, Department of Economics Working Paper).
- DINOPOULOS, E. and THOMPSON, P. (1996), "A Contribution to the Empirics of Endogenous Growth", *Eastern Economic Journal*, **22**, 389-400.
- DINOPOULOS, E. and THOMPSON, P. (1998), "Schumpeterian Growth without Scale Effects", *Journal of Economic Growth*, **3**, 313-335.
- DINOPOULOS, E. and THOMPSON, P. (1999), "Scale Effects in Schumpeterian Models of Economic Growth", *Journal of Evolutionary Economics*, **9**, 157-185.
- DIXIT, A. and STIGLITZ, J. E. (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review*, **67**, 297-308.
- DORNBUSCH, R., FISCHER S., and SAMUELSON P.A. (1977), "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods", *American Economic Review*, **67**, 823-839.
- ERIKSSON, C. and LINDH, T. (2000), "Growth Cycles with Technology Shifts and Externalities", *Economic Modelling*, **17**, 139-170.
- FREEMAN, C. (1982), *The Economics of Industrial Innovation* (Cambridge: MIT Press).
- GRILICHES, Z. (1957), "Hybrid Corn: An Exploration in the Economics of Technological Change", *Econometrica*, **25**, 501-522.
- GROSSMAN, G. M. and HELPMAN, E. (1990), "Comparative Advantage and Long-Run Growth", *American Economic Review*, **80**, 796-815.
- GROSSMAN, G. M. and HELPMAN, E. (1991a), "Endogenous Product Cycles", *Economic Journal*, **101**, 1214-1244.

- GROSSMAN, G. M. and HELPMAN, E. (1991b), "Quality Ladders and Product Cycles", *Quarterly Journal of Economics*, **425**, 557-586.
- GROSSMAN, G. M. and HELPMAN, E. (1991c), "Quality Ladders in the Theory of Growth", *Review of Economic Studies*, **59**, 43-61.
- HELPMAN, E. (1998) *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press).
- HELPMAN, E. and KRUGMAN P. R. (1989), *Trade Policy and Market Structure*. (Cambridge, MA: MIT Press).
- HELPMAN, E. and RANGEL, A. (1998), "Adjusting to a New Technology: Experience and Training", (NBER Working Paper No. 6551).
- HELPMAN, E. and TRAJTENBERG, M. (1998a), Diffusion of General Purpose Technologies. in E. Helpman (eds.), *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press).
- HELPMAN, E. and TRAJTENBERG, M. (1998b), "A Time to Sow and a Time to Reap: Growth: Based on General Purpose Technologies" in E. Helpman, (eds.), *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press).
- HOBIIJN, B. and JOVANOVIĆ, B. (2001), "The Information-Technology Revolution and the Stock Market: Evidence", *American Economic Review*, **91**, 1203-1220.
- HOWIT, P. (1998), "Measurement, Obsolescence and General Purpose Technologies", in Helpman, E. (eds.), *General Purpose Technologies and Economic Growth*, (Cambridge: MIT Press), 219-251.
- JONES, C. (1995a), "Time-series Tests of Endogenous Growth Models", *Quarterly Journal of Economics*, **110**, 495-525.
- JONES, C. (1995b), "R&D-Based Models of Economic Growth", *Journal of Political Economy*, **103**, 759-784.
- JORGENSEN, D. (2001), "Information Technology and the U.S. Economy", *American Economic Review*, **91**, 1-32.
- JOVANOVIĆ, B. and ROUSSEAU, P. (2001), "Vintage Organization Capital" (NBER Working Paper No. 8166).
- KAMIEN, M. and SCHWARTZ, N. (1972), "Timing of Innovations Under Rivalry", *Econometrica*, **40**, 43-60.

- KLEPPER, S. (1996), "Entry, Exit, Growth, and Innovation Over the Product Life Cycle", *The American Economic Review*, **86**, 562-83.
- KRUGMAN, P. R. (1987), "The Narrow Moving Band, the Dutch Disease and the Competitive Consequences of Mrs. Thatcher", *Journal of Development Economics*, **27**, 41-55.
- LINK, A. N. (1982), "A Disaggregated Analysis of Industrial R&D: Product Versus Process Innovation," in Devendra Sahal (ed.), *The Transfer and Utilization of Technical Knowledge*. Lexington, MA: Lexington Books.
- MULLIGAN, C. B. and SALA-I-MARTIN, X. (1991), "A Note on the Time-Elimination Method for Solving Recursive Economic Models" (NBER Working Paper No. 116).
- MULLIGAN, C. B. and SALA-I-MARTIN, X. (1992), "Transition Dynamics in Two-Sector Models of Endogenous Growth", *Quarterly Journal of Economics*, **107**, 739-73.
- PERETTO, P. (1998), "Technological Change and Population Growth", *Journal of Economic Growth*, **3**, 283-311.
- PERETTO, P. and SMULDERS, S. (1998), "Specialization Knowledge Dilution, and Scale Effects in an IO-Based Growth Model", Center for Economic Research, Tilburg University, Paper No. 9082.
- SCHERER, F. M. (1980), *Industrial market Structure on Economic Performance*, (Chicago: Rand McNally).
- SCHERER, F. M. (1991), "Changing Perspectives on the Firm Size Problem" in Zoltan J. Acs and David B. Audretsch (eds.), *Innovation and Technological Change: An International Comparison* (New York: Harvester Wheatsheaf).
- SCHUMPETER, J. A. (1934) *The Theory of Economic Development*, (Cambridge: Harvard University Press).
- SCHUMPETER, J. A. (1939) *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process* (New York: McGraw-Hill).
- SCHUMPETER, J. A. (1950) *Capitalist, Socialism, and Democracy*, (3rd edition) (New York: Harper and Brothers).
- SEGERSTROM, P. (1998), "Endogenous Growth without Scale Effects", *American Economic Review*, **88**, 1290-1310.

- SEGERSTROM, P. and ZOLNIEREK, J. (1999), "The R&D Incentives of Industry Leaders", *International Economic Review*, **40**, 745-766.
- TAYLOR, M. S. (1993), "'Quality Ladders' and Ricardian Trade", *Journal of International Economics*, **34**, 225-243.
- THIRTLY, C. and RUTTAN, V. (1987) *The Role of Demand and Supply in the Generation and Diffusion of Technical Change* (Switzerland: Harwood Academic Publishers).
- YOUNG, A. (1998), "Growth without Scale Effects", *Journal of Political Economy*, **106**, 41-63.

BIOGRAPHICAL SKETCH

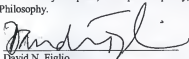
Iordanis Petsas was born in Thessaloniki, Greece, in 1974. He received his B.Sc. in economics from the Aristotle University of Thessaloniki in 1997. He then entered the doctoral program in the Economics Department of the University of Florida in August 1997. He received his M.A. in economics in 1999. Beginning in Fall 2002, he will join the Economics and Finance Department in the Kania School of Management at the University of Scranton, Scranton, Pennsylvania.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



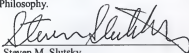
Elias Dinopoulos, Chairman
Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



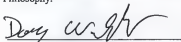
David N. Figlio
Associate Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



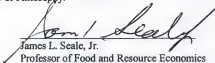
Steven M. Slutsky
Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Douglas G. Waldo
Associate Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



James L. Seale, Jr.
Professor of Food and Resource Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

August 2002

Dean, Graduate School

LD
1780
2002

,P498

